ICSE – Class IX Mathematics – M.L. Agarwal Solution

Chapter 1 : Rational and Irrational Numbers

EXERCISE 1.1

1. Insert a rational number between and 2/9 and 3/8 arrange in descending order.

Solution:

Given:

Rational numbers: 2/9 and 3/8

Let us rationalize the numbers,

By taking LCM for denominators 9 and 8 which is 72.

2/9 = (2×8)/(9×8) = 16/72

3/8 = (3×9)/(8×9) = 27/72

Since,

16/72 < 27/72

So, 2/9 < 3/8

The rational number between 2/9 and 3/8 is

$$= \frac{\frac{2}{9} + \frac{3}{8}}{2}$$

$$= \frac{\frac{(2\times 8)+(3\times 9)}{72}}{2}$$

$$= \frac{16 + 27}{72 \times 2}$$

$$= \frac{43}{144}$$

Hence, 3/8 > 43/144 > 2/9

The descending order of the numbers is 3/8, 43/144, 2/9

2. Insert two rational numbers between 1/3 and 1/4 and arrange in ascending order.

Solution:

Given:

The rational numbers 1/3 and ¼

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{1}{4}}{2}$$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2}$$

= 7/24

Now let us find the rational number between ¼ and 7/24

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{4} + \frac{7}{24}}{2}$$

$$= \frac{\frac{6+7}{24}}{2}$$

$$= \frac{13}{24 \times 2}$$

= 13/48

So,

The two rational numbers between 1/3 and ¼ are

7/24 and 13/48

Hence, we know that, 1/3 > 7/24 > 13/48 > ¼

The ascending order is as follows: ¼, 13/48, 7/24, 1/3

3. Insert two rational numbers between – 1/3 and – 1/2 and arrange in ascending order.

Solution:

Given:

The rational numbers -1/3 and -1/2

By taking LCM and rationalizing, we get

$$= \frac{\frac{-1}{3} + \frac{-1}{2}}{2}$$

$$= \frac{\frac{-2-3}{6}}{2}$$

$$= \frac{-5}{6 \times 2}$$

= -5/12

So, the rational number between -1/3 and -1/2 is -5/12

-1/3 > -5/12 > -1/2

Now, let us find the rational number between -1/3 and -5/12

By taking LCM and rationalizing, we get

$$= \frac{\frac{-1}{3} + \frac{-5}{12}}{2}$$

$$= \frac{\frac{-4-5}{12}}{2}$$

$$= \frac{\frac{-4-5}{12}}{2}$$

$$= \frac{-9}{12 \times 2}$$

= -9/24

= -3/8

So, the rational number between -1/3 and -5/12 is -3/8

-1/3 > -3/8 > -5/12

Hence, the two rational numbers between -1/3 and -1/2 are

-1/3 > -3/8 > -5/12 > -1/2

The ascending is as follows: -1/2, -5/12, -3/8, -1/3

4. Insert three rational numbers between 1/3 and 4/5, and arrange in descending order.

Solution:

Given:

The rational numbers 1/3 and 4/5

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{4}{5}}{2}$$

$$= \frac{\frac{5+12}{15}}{2}$$

$$= \frac{17}{15 \times 2}$$

= 17/30

So, the rational number between 1/3 and 4/5 is 17/30

1/3 < 17/30 < 4/5

Now, let us find the rational numbers between 1/3 and 17/30

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{17}{30}}{2}$$

$$= \frac{\frac{10+17}{30}}{2}$$

$$= \frac{27}{30 \times 2}$$

= 27/60

So, the rational number between 1/3 and 17/30 is 27/60

1/3 < 27/60 < 17/30

Now, let us find the rational numbers between 17/30 and 4/5

By taking LCM and rationalizing, we get

$$= \frac{\frac{17}{30} + \frac{4}{5}}{2}$$

$$= \frac{\frac{17+24}{30}}{2}$$

$$= \frac{41}{30 \times 2}$$

= 41/60

So, the rational number between 17/30 and 4/5 is 41/60

$17/30 < 41/60 < 4/5$

Hence, the three rational numbers between 1/3 and 4/5 are

$1/3 < 27/60 < 17/30 < 41/60 < 4/5$

The descending order is as follows: 4/5, 41/60, 17/30, 27/60, 1/3

5. Insert three rational numbers between 4 and 4.5.

Solution:

Given:

The rational numbers 4 and 4.5

By rationalizing, we get

$= (4 + 4.5)/2$

$= 8.5 / 2$

$= 4.25$

So, the rational number between 4 and 4.5 is 4.25

$4 < 4.25 < 4.5$

Now, let us find the rational number between 4 and 4.25

By rationalizing, we get

$= (4 + 4.25)/2$

$= 8.25 / 2$

$= 4.125$

So, the rational number between 4 and 4.25 is 4.125

$4 < 4.125 < 4.25$

Now, let us find the rational number between 4 and 4.125

By rationalizing, we get

$= (4 + 4.125)/2$

$= 8.125 / 2$

$= 4.0625$

So, the rational number between 4 and 4.125 is 4.0625

$4 < 4.0625 < 4.125$

Hence, the rational numbers between 4 and 4.5 are

$4 < 4.0625 < 4.125 < 4.25 < 4.5$

The three rational numbers between 4 and 4.5

4.0625, 4.125, 4.25

6. Find six rational numbers between 3 and 4.

Solution:

Given:

The rational number 3 and 4

So let us find the six rational numbers between 3 and 4,

First rational number between 3 and 4 is

= (3 + 4) / 2

= 7/2

Second rational number between 3 and 7/2 is

= (3 + 7/2) / 2

= (6+7) / (2 × 2) [By taking 2 as LCM]

= 13/4

Third rational number between 7/2 and 4 is

= (7/2 + 4) / 2

= (7+8) / (2 × 2) [By taking 2 as LCM]

= 15/4

Fourth rational number between 3 and 13/4 is

= (3 + 13/4) / 2

= (12+13) / (4 × 2) [By taking 4 as LCM]

= 25/8

Fifth rational number between 13/4 and 7/2 is

= [(13/4) + (7/2)] / 2

= [(13+14)/4] / 2 [By taking 4 as LCM]

= (13 + 14) / (4 × 2)

= 27/8

Sixth rational number between 7/2 and 15/4 is

= [(7/2) + (15/4)] / 2

= [(14 + 15)/4] / 2 [By taking 4 as LCM]

= (14 + 15) / (4 × 2)

= 29/8

Hence, the six rational numbers between 3 and 4 are

25/8, 13/4, 27/8, 7/2, 29/8, 15/4

7. Find five rational numbers between 3/5 and 4/5.

Solution:

Given:

The rational numbers 3/5 and 4/5

Now, let us find the five rational numbers between 3/5 and 4/5

So we need to multiply both numerator and denominator with 5 + 1 = 6

We get,

3/5 = (3 × 6) / (5 × 6) = 18/30

4/5 = (4 × 6) / (5 × 6) = 24/30

Now, we have 18/30 < 19/30 < 20/30 < 21/30 < 22/30 < 23/30 < 24/30

Hence, the five rational numbers between 3/5 and 4/5 are

19/30, 20/30, 21/30, 22/30, 23/30

8. Find ten rational numbers between -2/5 and 1/7.

Solution:

Given:

The rational numbers -2/5 and 1/7

By taking LCM for 5 and 7 which is 35

So, -2/5 = (-2 × 7) / (5 × 7) = -14/35

1/7 = (1 × 5) / (7 × 5) = 5/35

Now, we can insert any10 numbers between -14/35 and 5/35

i.e., -13/35, -12/35, -11/35, -10/35, -9/35, -8/35, -7/35, -6/35, -5/35, -4/35, -3/35, -2/35, -1/35, 1/35, 2/35, 3/35, 4/35

Hence, the ten rational numbers between -2/5 and 1/7 are

-6/35, -5/35, -4/35, -3/35, -2/35, -1/35, 1/35, 2/35, 3/35, 4/35

9. Find six rational numbers between 1/2 and 2/3.

Solution:

Given:

The rational number ½ and 2/3

To make the denominators similar let us take LCM for 2 and 3 which is 6

½ = (1 × 3) / (2 × 3) = 3/6

2/3 = (2 × 2) / (3 × 2) = 4/6

Now, we need to insert six rational numbers, so multiply both numerator and denominator by 6 + 1 = 7

3/6 = (3 × 7) / (6 × 7) = 21/42

4/6 = (4 × 7) / (6 × 7) = 28/42

We know that, 21/42 < 22/42 < 23/42 < 24/42 < 25/42 < 26/42 < 27/42 < 28/42

Hence, the six rational numbers between ½ and 2/3 are

22/42, 23/42, 24/42, 25/42, 26/42, 27/42

EXERCISE 1.2

1. Prove that, √5 is an irrational number.

Solution:

Let us consider √5 be a rational number, then

√5 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$5 = p^2 / q^2$

$p^2 = 5q^2$ …. (1)

As we know, '5' divides $5q^2$, so '5' divides p^2 as well. Hence, '5' is prime.

So 5 divides p

Now, let p = 5k, where 'k' is an integer

Square on both sides, we get

$p^2 = 25k^2$

$5q^2 = 25k^2$ [Since, $p^2 = 5q^2$, from equation (1)]

$q^2 = 5k^2$

As we know, '5' divides $5k^2$, so '5' divides q^2 as well. But '5' is prime.

So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts that 'p' and 'q' has no common factors (except 1).

We can say that, √5 is not a rational number.

√5 is an irrational number.

Hence proved.

2. Prove that, √7 is an irrational number.

Solution:

Let us consider √7 be a rational number, then

√7 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$7 = p^2 / q^2$

$p^2 = 7q^2$ …. (1)

As we know, ‘7’ divides $7q^2$, so ‘7’ divides p^2 as well. Hence, ‘7’ is prime.

So 7 divides p

Now, let p = 7k, where ‘k’ is an integer

Square on both sides, we get

$p^2 = 49k^2$

$7q^2 = 49k^2$ [Since, $p^2 = 7q^2$, from equation (1)]

$q^2 = 7k^2$

As we know, ‘7’ divides $7k^2$, so ‘7’ divides q^2 as well. But ‘7’ is prime.

So 7 divides q

Thus, p and q have a common factor 7. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, √7 is not a rational number.

√7 is an irrational number.

Hence proved.

3. Prove that √6 is an irrational number.

Solution:

Let us consider √6 be a rational number, then

√6 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$6 = p^2 / q^2$

$p^2 = 6q^2$ …. (1)

As we know, ‘2’ divides $6q^2$, so ‘2’ divides p^2 as well. Hence, ‘2’ is prime.

So 2 divides p

Now, let p = 2k, where ‘k’ is an integer

Square on both sides, we get

$p^2 = 4k^2$

$6q^2 = 4k^2$ [Since, $p^2 = 6q^2$, from equation (1)]

$3q^2 = 2k^2$

As we know, ‘2’ divides $2k^2$, so ‘2’ divides $3q^2$ as well.

‘2’ should either divide 3 or divide q^2.

But ‘2’ does not divide 3. ‘2’ divides q^2 so ‘2’ is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, √6 is not a rational number.

√6 is an irrational number.

Hence proved.

4. Prove that 1/√11 is an irrational number.

Solution:

Let us consider 1/√11 be a rational number, then

1/√11 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$1/11 = p^2 / q^2$

$q^2 = 11p^2$ …. (1)

As we know, ‘11’ divides $11p^2$, so ‘11’ divides q^2 as well. Hence, ‘11’ is prime.

So 11 divides q

Now, let q = 11k, where ‘k’ is an integer

Square on both sides, we get

$q^2 = 121k^2$

$11p^2 = 121k^2$ [Since, $q^2 = 11p^2$, from equation (1)]

$p^2 = 11k^2$

As we know, ‘11’ divides $11k^2$, so ‘11’ divides p^2 as well. But ‘11’ is prime.

So 11 divides p

Thus, p and q have a common factor 11. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, 1/√11 is not a rational number.

1/√11 is an irrational number.

Hence proved.

5. Prove that √2 is an irrational number. Hence show that 3 — √2 is an irrational.

Solution:

Let us consider √2 be a rational number, then

√2 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$2 = p^2 / q^2$

$p^2 = 2q^2$ …. (1)

As we know, '2' divides $2q^2$, so '2' divides p^2 as well. Hence, '2' is prime.

So 2 divides p

Now, let p = 2k, where 'k' is an integer

Square on both sides, we get

$p^2 = 4k^2$

$2q^2 = 4k^2$ [Since, $p^2 = 2q^2$, from equation (1)]

$q^2 = 2k^2$

As we know, '2' divides $2k^2$, so '2' divides q^2 as well. But '2' is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that 'p' and 'q' has no common factors (except 1).

We can say that, √2 is not a rational number.

√2 is an irrational number.

Now, let us assume 3 – √2 be a rational number, 'r'

So, 3 – √2 = r

3 – r = √2

We know that, 'r' is rational, '3- r' is rational, so '√2' is also rational.

This contradicts the statement that √2 is irrational.

So, 3- √2 is irrational number.

Hence proved.

6. Prove that, √3 is an irrational number. Hence, show that 2/5×√3 is an irrational number.

Solution:

Let us consider √3 be a rational number, then

√3 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$3 = p^2 / q^2$

$p^2 = 3q^2$ …. (1)

As we know, ‘3’ divides $3q^2$, so ‘3’ divides p^2 as well. Hence, ‘3’ is prime.

So 3 divides p

Now, let p = 3k, where ‘k’ is an integer

Square on both sides, we get

$p^2 = 9k^2$

$3q^2 = 9k^2$ [Since, $p^2 = 3q^2$, from equation (1)]

$q^2 = 3k^2$

As we know, ‘3’ divides $3k^2$, so ‘3’ divides q^2 as well. But ‘3’ is prime.

So 3 divides q

Thus, p and q have a common factor 3. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, √3 is not a rational number.

√3 is an irrational number.

Now, let us assume (2/5)√3 be a rational number, ‘r’

So, (2/5)√3 = r

5r/2 = √3

We know that, ‘r’ is rational, ‘5r/2’ is rational, so ‘√3’ is also rational.

This contradicts the statement that √3 is irrational.

So, (2/5)√3 is irrational number.

Hence proved.

7. Prove that √5 is an irrational number. Hence, show that -3 + 2√5 is an irrational number.

Solution:

Let us consider √5 be a rational number, then

√5 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$5 = p^2 / q^2$

$p^2 = 5q^2$ …. (1)

As we know, ‘5’ divides $5q^2$, so ‘5’ divides p^2 as well. Hence, ‘5’ is prime.

So 5 divides p

Now, let p = 5k, where ‘k’ is an integer

Square on both sides, we get

$p^2 = 25k^2$

$5q^2 = 25k^2$ [Since, $p^2 = 5q^2$, from equation (1)]

$q^2 = 5k^2$

As we know, ‘5’ divides $5k^2$, so ‘5’ divides q^2 as well. But ‘5’ is prime.

So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, √5 is not a rational number.

√5 is an irrational number.

Now, let us assume -3 + 2√5 be a rational number, ‘r’

So, -3 + 2√5 = r

-3 – r = 2√5

(-3 – r)/2 = √5

We know that, ‘r’ is rational, ‘(-3 – r)/2’ is rational, so ‘√5’ is also rational.

This contradicts the statement that √5 is irrational.

So, -3 + 2√5 is irrational number.

Hence proved.

8. Prove that the following numbers are irrational:

(i) 5 +√2

(ii) 3 – 5√3

(iii) 2√3 – 7

(iv) √2 +√5

Solution:

(i) 5 +√2

Now, let us assume 5 + √2 be a rational number, ‘r’

So, 5 + √2 = r

r – 5 = √2

We know that, ‘r’ is rational, ‘r – 5’ is rational, so ‘√2’ is also rational.

This contradicts the statement that √2 is irrational.

So, 5 + √2 is irrational number.

(ii) 3 – 5√3

Now, let us assume $3 - 5\sqrt{3}$ be a rational number, 'r'

So, $3 - 5\sqrt{3} = r$

$3 - r = 5\sqrt{3}$

$(3 - r)/5 = \sqrt{3}$

We know that, 'r' is rational, '$(3 - r)/5$' is rational, so '$\sqrt{3}$' is also rational.

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $3 - 5\sqrt{3}$ is irrational number.

(iii) $2\sqrt{3} - 7$

Now, let us assume $2\sqrt{3} - 7$ be a rational number, 'r'

So, $2\sqrt{3} - 7 = r$

$2\sqrt{3} = r + 7$

$\sqrt{3} = (r + 7)/2$

We know that, 'r' is rational, '$(r + 7)/2$' is rational, so '$\sqrt{3}$' is also rational.

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $2\sqrt{3} - 7$ is irrational number.

(iv) $\sqrt{2} + \sqrt{5}$

Now, let us assume $\sqrt{2} + \sqrt{5}$ be a rational number, 'r'

So, $\sqrt{2} + \sqrt{5} = r$

$\sqrt{5} = r - \sqrt{2}$

Square on both sides,

$(\sqrt{5})^2 = (r - \sqrt{2})^2$

$5 = r^2 + (\sqrt{2})^2 - 2r\sqrt{2}$

$5 = r^2 + 2 - 2\sqrt{2}r$

$5 - 2 = r^2 - 2\sqrt{2}r$

$r^2 - 3 = 2\sqrt{2}r$

$(r^2 - 3)/2r = \sqrt{2}$

We know that, 'r' is rational, '$(r^2 - 3)/2r$' is rational, so '$\sqrt{2}$' is also rational.

This contradicts the statement that $\sqrt{2}$ is irrational.

So, $\sqrt{2} + \sqrt{5}$ is irrational number.

EXERCISE 1.3

1. Locate $\sqrt{10}$ and $\sqrt{17}$ on the amber line.

Solution:

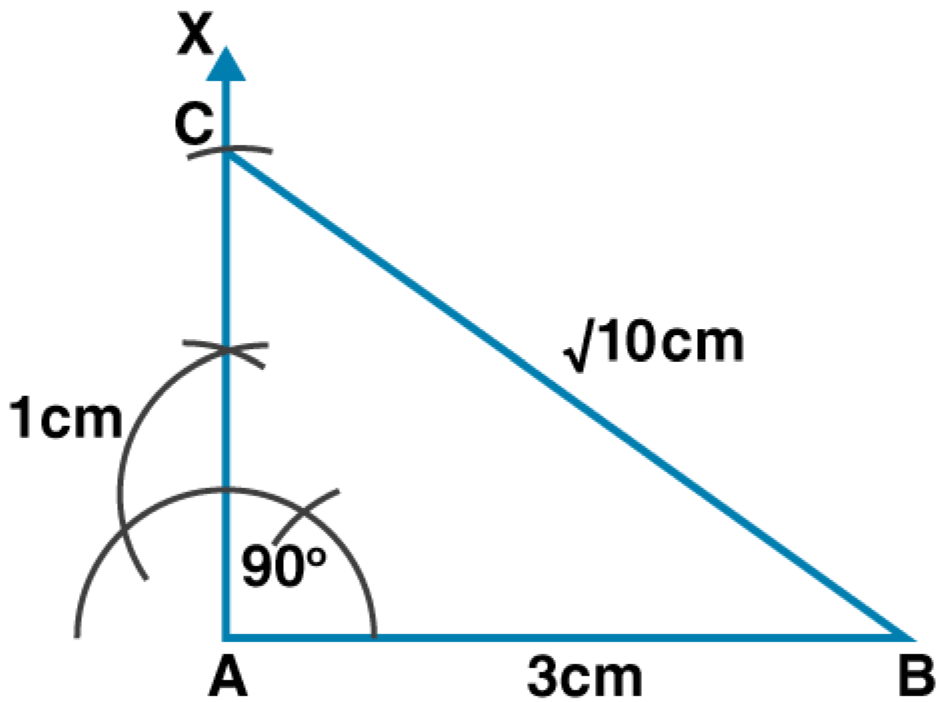
√10

$\sqrt{10} = \sqrt{(9 + 1)} = \sqrt{((3)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB = 3cm.
- At point A, draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

BC = √10cm



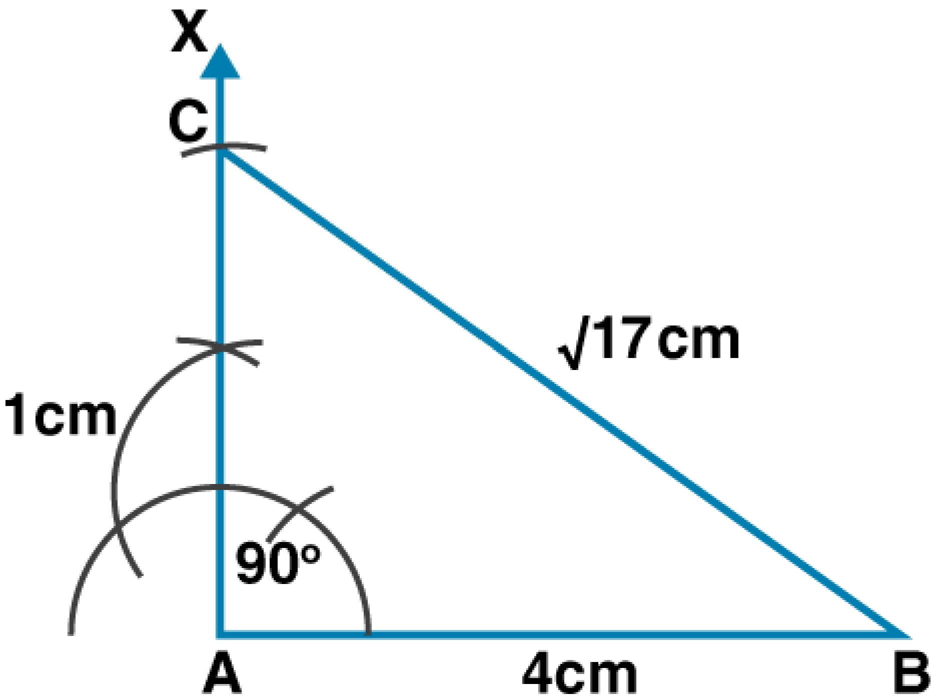
√17

$\sqrt{17} = \sqrt{(16 + 1)} = \sqrt{((4)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB = 4cm.
- At point A, draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

BC = √17cm



2. Write the decimal expansion of each of the following numbers and say what kind of decimal expansion each has:

(i) 36/100

(ii) 4 1/8

(iii) 2/9

(iv) 2/11

(v) 3/13

(vi) 329/400

Solution:

(i) 36/100

			0	0.	3	6	0
1	0	0	3	6.	0	0	0
		–	0				
			3	6			
		–		0			
			3	6	0		
		–	3	0	0		
				6	0	0	
			–	6	0	0	
						0	0
					–		0
							0

36/100 = 0.36

It is a terminating decimal.

(ii) 4 1/8

4 1/8 = (4×8 + 1)/8 = 33/8

	0	4.	1	2	5
8	3	3.	0	0	0
–	0				
	3	3			
–	3	2			
		1	0		
	–		8		
			2	0	
		–	1	6	
				4	0
			–	4	0
					0

33/8 = 4.125

It is a terminating decimal.

(iii) 2/9

	0.	2	2	2
9	2.	0	0	0
−	0			
	2	0		
−	1	8		
		2	0	
	−	1	8	
			2	0
		−	1	8
				2

2/9 = 0.222

It is a non-terminating recurring decimal.

(iv) 2/11

		0.	1	8	1
1	1	2.	0	0	0
	−	0			
		2	0		
	−	1	1		
			9	0	
		−	8	8	
				2	0
			−	1	1
					9

2/11 = 0.181

It is a non-terminating recurring decimal.

(v) 3/13

		0.	2	3	0	7	6	9	2	3	0	7
1	3	3.	0	0	0	0	0	0	0	0	0	0
	−	0										
		3	0									
	−	2	6									
			4	0								
		−	3	9								
				1	0							
			−		0							
				1	0	0						
			−		9	1						
						9	0					
					−	7	8					
						1	2	0				
					−	1	1	7				
								3	0			
							−	2	6			
									4	0		
								−	3	9		
										1	0	
									−		0	
										1	0	0
									−		9	1
												9

3/13 = 0.2317692307

It is a non-terminating recurring decimal.

(vi) 329/400

			0	0	0.	8	2	2	5
4	0	0	3	2	9.	0	0	0	0
		−	0						
			3	2					
		−		0					
			3	2	9				
		−			0				
			3	2	9	0			
		−	3	2	0	0			
					9	0	0		
				−	8	0	0		
					1	0	0	0	
				−		8	0	0	
						2	0	0	0
					−	2	0	0	0
									0

329/400 = 0.8225

It is a terminating decimal.

3. Without actually performing the king division, State whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) 13/3125

(ii) 17/8

(iii) 23/75

(iv) 6/15

(v) 1258/625

(vi) 77/210

Solution:

We know that, if the denominator of a fraction has only 2 or 5 or both factors, it is a terminating decimal otherwise it is non-terminating repeating decimals.

(i) 13/3125

5	3125
5	625
5	125
5	25
5	5
	1

3125 = 5 × 5 × 5 × 5 × 5

Prime factor of 3125 = 5, 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

(ii) 17/8

2	8
2	4
2	2
	1

8 = 2 × 2 × 2

Prime factor of 8 = 2, 2, 2 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

(iii) 23/75

3	75
5	25
5	5
	1

75 = 3 × 5 × 5

Prime factor of 75 = 3, 5, 5

It is a non-terminating repeating decimal.

(iv) 6/15

Let us divide both numerator and denominator by 3

6/15 = (6 ÷ 3) / (15 ÷ 3)

= 2/5

Since the denominator is 5.

It is a terminating decimal.

(v) 1258/625

5	625
5	125
5	25
5	5
	1

625 = 5 × 5 × 5 × 5

Prime factor of 625 = 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

(vi) 77/210

Let us divide both numerator and denominator by 7

77/210 = (77 ÷ 7) / (210 ÷ 7)

= 11/30

2	30
3	15
5	5
	1

30 = 2 × 3 × 5

Prime factor of 30 = 2, 3, 5

It is a non-terminating repeating decimal.

4. Without actually performing the long division, find if 987/10500 will have terminating or non-terminating repeating decimal expansion. Give reasons for your answer.

Solution:

Given:

The fraction 987/10500

Let us divide numerator and denominator by 21, we get

987/10500 = (987 ÷ 21) / (10500 ÷ 21)

= 47/500

So,

The prime factors for denominator 500 = 2 × 2 × 5 × 5 × 5

Since it is of the form: 2^n, 5^n

Hence it is a terminating decimal.

5. Write the decimal expansions of the following numbers which have terminating decimal expansions:

(i) 17/8

(ii) 13/3125

(iii) 7/80

(iv) 6/15

(v) $2^2 \times 7/5^4$

(vi) 237/1500

Solution:

(i) 17/8

2	8
2	4
2	2
	1

Denominator, 8 = 2 × 2 × 2

= 2^3

It is a terminating decimal.

When we divide 17/8, we get

	0	2.	1	2	5	0
8	1	7.	0	0	0	0
−	0					
	1	7				
−	1	6				
		1	0			
	−		8			
			2	0		
		−	1	6		
				4	0	
			−	4	0	
					0	0
				−		0
						0

17/8 = 2.125

(ii) 13/3125

5	3125
5	625
5	125
5	25
5	5
	1

3125 = 5 × 5 × 5 × 5 × 5

Prime factor of 3125 = 5, 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

When we divide 13/3125, we get

				0	0.	0	0	4	1	6
3	1	2	5	1	3.	0	0	0	0	0
			−	0						
				1	3					
			−		0					
				1	3	0				
			−			0				
				1	3	0	0			
			−				0			
				1	3	0	0	0		
			−	1	2	5	0	0		
						5	0	0	0	
					−	3	1	2	5	
						1	8	7	5	0
					−	1	8	7	5	0
										0

13/3125 = 0.00416

(iii) 7/80

2	80
2	40
2	20
2	10
5	5
	1

80 = 2 × 2 × 2 × 2 × 5

Prime factor of 80 = 2^4, 5^1 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

When we divide 7/80, we get

		0.	0	8	7	5
8	0	7.	0	0	0	0
	−	0				
		7	0			
	−		0			
		7	0	0		
	−	6	4	0		
			6	0	0	
		−	5	6	0	
				4	0	0
			−	4	0	0
						0

7/80 = 0.0875

(iv) 6/15

Let us divide both numerator and denominator by 3, we get

6/15 = (6 ÷ 3) / (15 ÷ 3)

= 2/5

Since the denominator is 5,

It is terminating decimal.

		0.	4	0
1	5	6.	0	0
	−	0		
		6	0	
	−	6	0	
			0	0
		−		0
				0

6/15 = 0.4

(v) $(2^2 \times 7)/5^4$

We know that the denominator is 5^4

It is a terminating decimal.

$(2^2 \times 7)/5^4 = (2 \times 2 \times 7) / (5 \times 5 \times 5 \times 5)$

$= 28/625$

			0	0.	0	4	4	8
6	2	5	2	8.	0	0	0	0
		−	0					
			2	8				
		−		0				
			2	8	0			
		−			0			
			2	8	0	0		
		−	2	5	0	0		
				3	0	0	0	
			−	2	5	0	0	
					5	0	0	0
				−	5	0	0	0
								0

$28/625 = 0.0448$

It is a terminating decimal.

(vi) $237/1500$

Let us divide both numerator and denominator by 3, we get

$237/1500 = (237 \div 3) / (1500 \div 3)$

$= 79/500$

Since the denominator is 500,

Its factors are, $500 = 2 \times 2 \times 5 \times 5 \times 5$

$= 2^2 \times 5^3$

It is terminating decimal.

			0	0.	1	5	8
5	0	0	7	9.	0	0	0
		−	0				
			7	9			
		−		0			
			7	9	0		
		−	5	0	0		
			2	9	0	0	
		−	2	5	0	0	
				4	0	0	0
			−	4	0	0	0
							0

237/1500= 79/500 = 0.1518

6. Write the denominator of the rational number 257/5000 in the form $2^m \times 5^n$ where m, n is non-negative integers. Hence, write its decimal expansion on without actual division.

Solution:

Given:

The fraction 257/5000

Since the denominator is 5000,

The factors for 5000 are:

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

5000 = 2 × 2 × 2 × 5 × 5 × 5 × 5

= $2^3 \times 5^4$

257/5000 = 257/($2^3 \times 5^4$)

It is a terminating decimal.

So,

Let us multiply both numerator and denominator by 2, we get

257/5000 = (257 × 2) / (5000 × 2)

= 514/10000

= 0.0514

7. Write the decimal expansion of 1/7. Hence, write the decimal expression of? 2/7, 3/7, 4/7, 5/7 and 6/7.

Solution:

Given:

The fraction: 1/7

	0.	1	4	2	8	5	7	1	4	2	8	5	7
7	1.	0	0	0	0	0	0	0	0	0	0	0	0
−	0												
	1	0											
−		7											
		3	0										
	−	2	8										
			2	0									
		−	1	4									
				6	0								
			−	5	6								
					4	0							
				−	3	5							
						5	0						
					−	4	9						
							1	0					
						−		7					
								3	0				
							−	2	8				
									2	0			
								−	1	4			
										6	0		
									−	5	6		
											4	0	
										−	3	5	
												5	0
											−	4	9
													1

1/7 = 0.142857142857

Since it is recurring,

$= 0.\overline{142857}$

Now,

$2/7 = 2 \times (1/7)$

$= 2 \times 0.\overline{142857}$

$= 0.\overline{285714}$

$3/7 = 3 \times (1/7)$

$= 3 \times 0.\overline{142857}$

$= 0.\overline{428571}$

$4/7 = 4 \times (1/7)$

$= 4 \times 0.\overline{142857}$

$= 0.\overline{571428}$

$5/7 = 5 \times (1/7)$

$= 5 \times 0.\overline{142857}$

$= 0.\overline{714285}$

$6/7 = 6 \times (1/7)$

$= 6 \times 0.\overline{142857}$

$= 0.\overline{857142}$

8. Express the following numbers in the form p/q'. Where p and q are both integers and q≠0;

$(i) 0.\overline{3}$

$(ii) 5.\overline{2}$

$(iii) 0.404040....$

$(iv) 0.4\overline{7}$

$(v) 0.1\overline{34}$

$(vi) 0.\overline{001}$

Solution:

$(i) 0.\overline{3}$

Let x = $0.\overline{3}$ = 0.3333…

Since there is one repeating digit after the decimal point, Multiplying by 10 on both sides, we get

10x = 3.3333…

Now, subtract both the values,

9x = 3

x = 3/9

= 1/3

$0.\overline{3}$ = 1/3

$(ii) 5.\overline{2}$

Let x = $5.\overline{2}$ = 5.2222…

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

10x = 52.2222…

Now, subtract both the values,

9x = 52 – 5

9x = 47

x = 47/9

$5.\overline{2}$ = 47/9

$(iii) 0.404040....$

Let x = 0.404040

Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get

100x = 40.404040…

Now, subtract both the values,

99x = 40

x = 40/99

0.404040… = 40/99

$(iv) 0.4\overline{7}$

Let x = $0.4\overline{7}$ = 0.47777…

Since there is one non-repeating digit after the decimal point, Multiplying by 10 on both sides, we get

10x = 4.7777

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

100x = 47.7777

Now, subtract both the values,

90x = 47 – 4

90x = 43

x = 43/90

$\mathbf{0.4\overline{7}}$ = 43/90

$(v)\mathbf{0.1\overline{34}}$

Let x = $\mathbf{0.1\overline{34}}$ = 0.13434343…

Since there is one non-repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

10x = 1.343434

Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get

1000x = 134.343434

Now, subtract both the values,

990x = 133

x = 133/990

$\mathbf{0.1\overline{34}}$ = 133/990

$(vi)\mathbf{0.\overline{001}}$

Let x = $\mathbf{0.\overline{001}}$ = 0.001001001…

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

1000x = 1.001001

Now, subtract both the values,

999x = 1

x = 1/999

$0.\overline{001}$ = 1/999

9. Classify the following numbers as rational or irrational:

(i) √23

(ii) √225

(iii) 0.3796

(iv) 7.478478

(v) 1.101001000100001…

$(vi) 345.0\overline{456}$

Solution:

(i) √23

Since, 23 is not a perfect square,

√23 is an irrational number.

(ii) √225

√225 = $\sqrt{(15)^2}$ = 15

Since, 225 is a perfect square,

√225 is a rational number.

(iii) 0.3796

0.3796 = 3796/1000

Since, the decimal expansion is terminating decimal.

0.3796 is a rational number.

(iv) 7.478478

Let x = 7.478478

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

1000x = 7478.478478…

Now, subtract both the values,

999x = 7478 – 7

999x = 7471

x = 7471/999

7.478478 = 7471/999

Hence, it is neither terminating nor non-terminating or non-repeating decimal.

7.478478 is an irrational number.

(v) 1.101001000100001…

Since number of zero's between two consecutive ones are increasing. So it is non-terminating or non-repeating decimal.

1.101001000100001… is an irrational number.

$(vi) 345.0\overline{456}$

Let x = 345.0456456

Multiplying by 10 on both sides, we get

10x = 3450.456456

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

1000x = 3450456.456456…

Now, subtract both the values,

10000x – 10x = 3450456 – 345

9990x = 3450111

x = 3450111/9990

Since, it is non-terminating repeating decimal.

$345.0\overline{456}$ is a rational number.

10. The following real numbers have decimal expansions as given below. In each case, state whether they are rational or not. If they are rational and expressed in the form p/q, where p, q are integers, q≠ 0 and p, q are co-prime, then what can you say about the prime factors of q?

(i) 37.09158

(ii) $423.\overline{04567}$

(iii) 8.9010010001…

(iv) 2.3476817681…

Solution:

(i) 37.09158

We know that

It has terminating decimal

Here

It is a rational number and factors of q will be 2 or 5 or both.

(ii) $423.\overline{04567}$

We know that

It has non-terminating recurring decimals

Here

It is a rational number.

(iii) 8.9010010001…

We know that

It has non-terminating, non-recurring decimal.

Here

It is not a rational number.

(iv) 2.3476817681…

We know that

It has non-terminating, recurring decimal.

Here

It is a rational number and the factors of q are prime factors other than 2 and 5.

11. Insert an irrational number between the following.

(i) 1/3 and ½

(ii) -2/5 and ½

(iii) 0 and 0.1

Solution:

(i) One irrational number between 1/3 and ½

	0.	3	3	3
3	1.	0	0	0
−	0			
	1	0		
−		9		
		1	0	
	−		9	
			1	0
		−		9
				1

1/3 = 0.333…

	0.	5
2	1.	0
−	0	
	1	0
−	1	0
		0

½ = 0.5

So there are infinite irrational numbers between 1/3 and ½.

One irrational number among them can be 0.4040040004…

(ii) One irrational number between -2/5 and ½

		-	0.	4
+	5	-	2.	0
	−		0	
			2	0
		−	2	0
				0

-2/5 = -0.4

	0.	5
2	1.	0
−	0	
	1	0
−	1	0
		0

½ = 0.5

So there are infinite irrational numbers between -2/5 and ½.

One irrational number among them can be 0.1010010001…

(iii) One irrational number between 0 and 0.1

There are infinite irrational numbers between 0 and 1.

One irrational number among them can be 0.06006000600006…

12. Insert two irrational numbers between 2 and 3.

Solution:

2 is expressed as √4

And 3 is expressed as √9

So, two irrational numbers between 2 and 3 or √4 and √9 are √5, √6

13. Write two irrational numbers between 4/9 and 7/11.

Solution:

4/9 is expressed as 0.4444…

7/11 is expressed as 0.636363…

So, two irrational numbers between 4/9 and 7/11 are 0.4040040004… and 0.6060060006…

14. Find one rational number between √2 and √3.

Solution:

√2 is expressed as 1.4142…

√3 is expressed as 1.7320…

So, one rational number between √2 and √3 is 1.5.

15. Find two rational numbers between 2√3 and √15.

Solution:

√12 = √(4×3) = 2√3

Since, 12 < 12.25 < 12.96 < 15

So, √12 < √12.25 < √12.96 < √15

Hence, two rational numbers between √12 and √15are [√12.25, √12.96] or [√3.5, √3.6].

16. Insert irrational numbers between √5 and √7.

Solution:

Since, 5 < 6 < 7

So, irrational number between √5 and √7 is √6.

17. Insert two irrational numbers between √3 and √7.

Solution:

Since, 3 < 4 < 5 < 6 < 7

So,

√3 < √4 < √5 < √6 < √7

But √4 = 2, which is a rational number.

So,

Two irrational numbers between √3 and √7 are √5 and √6.

EXERCISE 1.4

1. Simplify the following:

(i) √45 – 3√20 + 4√5

(ii) 3√3 + 2√27 + 7/√3

(iii) 6√5 × 2√5

(iv) 8√15 ÷ 2√3

(v) √24/8 + √54/9

(vi) 3/√8 + 1/√2

Solution:

(i) √45 – 3√20 + 4√5

Let us simplify the expression,

√45 – 3√20 + 4√5

= √(9×5) – 3√(4×5) + 4√5

= 3√5 – 3×2√5 + 4√5

= 3√5 – 6√5 + 4√5

= √5

(ii) $3\sqrt{3} + 2\sqrt{27} + 7/\sqrt{3}$

Let us simplify the expression,

$3\sqrt{3} + 2\sqrt{27} + 7/\sqrt{3}$

$= 3\sqrt{3} + 2\sqrt{(9\times3)} + 7\sqrt{3}/(\sqrt{3}\times\sqrt{3})$ (by rationalizing)

$= 3\sqrt{3} + (2\times3)\sqrt{3} + 7\sqrt{3}/3$

$= 3\sqrt{3} + 6\sqrt{3} + (7/3)\sqrt{3}$

$= \sqrt{3}\,(3 + 6 + 7/3)$

$= \sqrt{3}\,(9 + 7/3)$

$= \sqrt{3}\,(27+7)/3$

$= 34/3\,\sqrt{3}$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

Let us simplify the expression,

$6\sqrt{5} \times 2\sqrt{5}$

$= 12 \times 5$

$= 60$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

Let us simplify the expression,

$8\sqrt{15} \div 2\sqrt{3}$

$= (8\sqrt{5}\sqrt{3}) / 2\sqrt{3}$

$= 4\sqrt{5}$

(v) $\sqrt{24}/8 + \sqrt{54}/9$

Let us simplify the expression,

$\sqrt{24}/8 + \sqrt{54}/9$

$= \sqrt{(4\times6)}/8 + \sqrt{(9\times6)}/9$

$= 2\sqrt{6}/8 + 3\sqrt{6}/9$

$= \sqrt{6}/4 + \sqrt{6}/3$

By taking LCM

$= (3\sqrt{6} + 4\sqrt{6})/12$

$= 7\sqrt{6}/12$

(vi) $3/\sqrt{8} + 1/\sqrt{2}$

Let us simplify the expression,

$3/\sqrt{8} + 1/\sqrt{2}$

$= 3/2\sqrt{2} + 1/\sqrt{2}$

By taking LCM

$= (3 + 2)/(2\sqrt{2})$

$= 5/(2\sqrt{2})$

By rationalizing,

$= 5\sqrt{2}/(2\sqrt{2} \times 2\sqrt{2})$

$= 5\sqrt{2}/(2\times2)$

$= 5\sqrt{2}/4$

2. Simplify the following:

(i) $(5 + \sqrt{7}) (2 + \sqrt{5})$

(ii) $(5 + \sqrt{5}) (5 – \sqrt{5})$

(iii) $(\sqrt{5} + \sqrt{2})^2$

(iv) $(\sqrt{3} – \sqrt{7})^2$

(v) $(\sqrt{2} + \sqrt{3}) (\sqrt{5} + \sqrt{7})$

(vi) $(4 + \sqrt{5}) (\sqrt{3} – \sqrt{7})$

Solution:

(i) $(5 + \sqrt{7}) (2 + \sqrt{5})$

Let us simplify the expression,

$= 5(2 + \sqrt{5}) + \sqrt{7}(2 + \sqrt{5})$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(ii) $(5 + \sqrt{5}) (5 – \sqrt{5})$

Let us simplify the expression,

By using the formula,

$(a)^2 – (b)^2 = (a + b) (a – b)$

So,

$= (5)^2 – (\sqrt{5})^2$

$= 25 – 5$

$= 20$

(iii) $(\sqrt{5} + \sqrt{2})^2$

Let us simplify the expression,

By using the formula,

$(a + b)^2 = a^2 + b^2 + 2ab$

$(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2\sqrt{5}\sqrt{2}$

$= 5 + 2 + 2\sqrt{10}$

$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{3} - \sqrt{7})^2$

Let us simplify the expression,

By using the formula,

$(a - b)^2 = a^2 + b^2 - 2ab$

$(\sqrt{3} - \sqrt{7})^2 = (\sqrt{3})^2 + (\sqrt{7})^2 - 2\sqrt{3}\sqrt{7}$

$= 3 + 7 - 2\sqrt{21}$

$= 10 - 2\sqrt{21}$

(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$

Let us simplify the expression,

$= \sqrt{2}(\sqrt{5} + \sqrt{7}) + \sqrt{3}(\sqrt{5} + \sqrt{7})$

$= \sqrt{2}\times\sqrt{5} + \sqrt{2}\times\sqrt{7} + \sqrt{3}\times\sqrt{5} + \sqrt{3}\times\sqrt{7}$

$= \sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$

(vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$

Let us simplify the expression,

$= 4(\sqrt{3} - \sqrt{7}) + \sqrt{5}(\sqrt{3} - \sqrt{7})$

$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$

3. If $\sqrt{2} = 1.414$, then find the value of

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$

Solution:

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Let us simplify the expression,

$\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

$= \sqrt{(2\times4)} + \sqrt{(2\times25)} + \sqrt{(2\times36)} + \sqrt{(2\times49)}$

$= \sqrt{2}\,\sqrt{4} + \sqrt{2}\,\sqrt{25} + \sqrt{2}\,\sqrt{36} + \sqrt{2}\,\sqrt{49}$

$= 2\sqrt{2} + 5\sqrt{2} + 6\sqrt{2} + 7\sqrt{2}$

= 20√2

= 20 × 1.414

= 28.28

(ii) 3√32 – 2√50 + 4√128 – 20√18

Let us simplify the expression,

3√32 – 2√50 + 4√128 – 20√18

= 3√(16×2) – 2√(25×2) + 4√(64×2) – 20√(9×2)

= 3√16 √2 – 2√25 √2 + 4√64 √2 – 20√9 √2

= 3.4√2 – 2.5√2 + 4.8√2 – 20.3√2

= 12√2 – 10√2 + 32√2 – 60√2

= (12 – 10 + 32 – 60) √2

= -26√2

= -26 × 1.414

= -36.764

4. If √3 = 1.732, then find the value of

(i) √27 + √75 + √108 – √243

(ii) 5√12 – 3√48 + 6√75 + 7√108

Solution:

(i) √27 + √75 + √108 – √243

Let us simplify the expression,

√27 + √75 + √108 – √243

= √(9×3) + √(25×3) + √(36×3) – √(81×3)

= √9 √3 + √25 √3 + √36 √3 – √81 √3

= 3√3 + 5√3 + 6√3 – 9√3

= (3 + 5 + 6 – 9) √3

= 5√3

= 5 × 1.732

= 8.660

(ii) 5√12 – 3√48 + 6√75 + 7√108

Let us simplify the expression,

5√12 – 3√48 + 6√75 + 7√108

= 5√(4×3) – 3√(16×3) + 6√(25×3) + 7√(36×3)

= 5√4 √3 – 3√16 √3 + 6√25 √3 + 7√36 √3

= 5.2√3 – 3.4√3 + 6.5√3 + 7.6√3

= 10√3 – 12√3 + 30√3 + 42√3

= (10 – 12 + 30 + 42) √3

= 70√3

= 70 × 1.732

= 121.24

5. State which of the following are rational or irrational decimals.

(i) √(4/9), -3/70, √(7/25), √(16/5)

(ii) -√(2/49), 3/200, √(25/3), -√(49/16)

Solution:

(i) √(4/9), -3/70, √(7/25), √(16/5)

√(4/9) = 2/3

-3/70 = -3/70

√(7/25) = √7/5

√(16/5) = 4/√5

So,

√7/5 and 4/√5 are irrational decimals.

2/3 and -3/70 are rational decimals.

(ii) -√(2/49), 3/200, √(25/3), -√(49/16)

-√(2/49) = -√2/7

3/200 = 3/200

√(25/3) = 5/√3

-√(49/16) = -7/4

So,

-√2/7 and 5/√3 are irrational decimals.

3/200 and -7/4 are rational decimals.

6. State which of the following are rational or irrational decimals.

(i) -3√2

(ii) √(256/81)

(iii) √(27×16)

(iv) √(5/36)

Solution:

(i) -3√2

We know that √2 is an irrational number.

So, -3√2 will also be irrational number.

(ii) √(256/81)

√(256/81) = 16/9 = 4/3

It is a rational number.

(iii) √(27×16)

√(27×16) = √(9×3×16) = 3×4√3 = 12√3

It is an irrational number.

(iv) √(5/36)

√(5/36) = √5/6

It is an irrational number.

7. State which of the following are irrational numbers.

(i) 3 – √(7/25)

(ii) -2/3 + ∛2

(iii) 3/√3

(iv) -2/7 ∛5

(v) (2 – √3) (2 + √3)

(vi) $(3 + \sqrt{5})^2$

(vii) $(2/5\ \sqrt{7})^2$

(viii) $(3 - \sqrt{6})^2$

Solution:

(i) 3 – √(7/25)

Let us simplify,

3 – √(7/25) = 3 – √7/√25

= 3 – √7/5

Hence, 3 – √7/5 is an irrational number.

(ii) -2/3 + ∛2

Let us simplify,

$-2/3 + \sqrt[3]{2} = -2/3 + 2^{1/3}$

Since, 2 is not a perfect cube.

Hence it is an irrational number.

(iii) $3/\sqrt{3}$

Let us simplify,

By rationalizing, we get

$3/\sqrt{3} = 3\sqrt{3} /(\sqrt{3}\times\sqrt{3})$

$= 3\sqrt{3}/3$

$= \sqrt{3}$

Hence, $3/\sqrt{3}$ is an irrational number.

(iv) $-2/7 \sqrt[3]{5}$

Let us simplify,

$-2/7 \sqrt[3]{5} = -2/7 (5)^{1/3}$

Since, 5 is not a perfect cube.

Hence it is an irrational number.

(v) $(2 - \sqrt{3}) (2 + \sqrt{3})$

Let us simplify,

By using the formula,

$(a + b) (a - b) = (a)^2 (b)^2$

$(2 - \sqrt{3}) (2 + \sqrt{3}) = (2)^2 - (\sqrt{3})^2$

$= 4 - 3$

$= 1$

Hence, it is a rational number.

(vi) $(3 + \sqrt{5})^2$

Let us simplify,

By using $(a + b)^2 = a^2 + b^2 + 2ab$

$(3 + \sqrt{5})^2 = 3^2 + (\sqrt{5})^2 + 2.3.\sqrt{5}$

$= 9 + 5 + 6\sqrt{5}$

$= 14 + 6\sqrt{5}$

Hence, it is an irrational number.

(vii) $(2/5\ \sqrt{7})^2$

Let us simplify,

$(2/5\ \sqrt{7})^2 = (2/5\ \sqrt{7}) \times (2/5\ \sqrt{7})$

$= 4/25 \times 7$

$= 28/25$

Hence it is a rational number.

(viii) $(3 - \sqrt{6})^2$

Let us simplify,

By using $(a - b)^2 = a^2 + b^2 - 2ab$

$(3 - \sqrt{6})^2 = 3^2 + (\sqrt{6})^2 - 2.3.\sqrt{6}$

$= 9 + 6 - 6\sqrt{6}$

$= 15 - 6\sqrt{6}$

Hence it is an irrational number.

8. Prove the following are irrational numbers.

(i) $\sqrt[3]{2}$

(ii) $\sqrt[3]{3}$

(iii) $\sqrt[4]{5}$

Solution:

(i) $\sqrt[3]{2}$

We know that $\sqrt[3]{2} = 2^{1/3}$

Let us consider $2^{1/3} = p/q$, where p, q are integers, q>0.

p and q have no common factors (except 1).

So,

$2^{1/3} = p/q$

$2 = p^3/q^3$

$p^3 = 2q^3$ ….. (1)

We know that, 2 divides $2q^3$ then 2 divides p^3

So, 2 divides p

Now, let us consider p = 2k, where k is an integer

Substitute the value of p in (1), we get

$p^3 = 2q^3$

$(2k)^3 = 2q^3$

$8k^3 = 2q^3$

$4k^3 = q^3$

We know that, 2 divides $4k^3$ then 2 divides q^3

So, 2 divides q

Thus p and q have a common factor '2'.

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[3]{2}$ is an irrational number.

(ii) $\sqrt[3]{3}$

We know that $\sqrt[3]{3} = 3^{1/3}$

Let us consider $3^{1/3} = p/q$, where p, q are integers, $q>0$.

p and q have no common factors (except 1).

So,

$3^{1/3} = p/q$

$3 = p^3/q^3$

$p^3 = 3q^3$ ….. (1)

We know that, 3 divides $3q^3$ then 3 divides p^3

So, 3 divides p

Now, let us consider p = 3k, where k is an integer

Substitute the value of p in (1), we get

$p^3 = 3q^3$

$(3k)^3 = 3q^3$

$9k^3 = 3q^3$

$3k^3 = q^3$

We know that, 3 divides $9k^3$ then 3 divides q^3

So, 3 divides q

Thus p and q have a common factor '3'.

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[3]{3}$ is an irrational number.

(iii) $\sqrt[4]{5}$

We know that $\sqrt[4]{5} = 5^{1/4}$

Let us consider $5^{1/4} = p/q$, where p, q are integers, q>0.

p and q have no common factors (except 1).

So,

$5^{1/4} = p/q$

$5 = p^4/q^4$

$P^4 = 5q^4$ ….. (1)

We know that, 5 divides $5q^4$ then 5 divides p^4

So, 5 divides p

Now, let us consider p = 5k, where k is an integer

Substitute the value of p in (1), we get

$P^4 = 5q^4$

$(5k)^4 = 5q^4$

$625k^4 = 5q^4$

$125k^4 = q^4$

We know that, 5 divides $125k^4$ then 5 divides q^4

So, 5 divides q

Thus p and q have a common factor '5'.

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[4]{5}$ is an irrational number.

9. Find the greatest and the smallest real numbers.

(i) 2√3, 3/√2, -√7, √15

(ii) -3√2, 9/√5, -4, 4/3 √5, 3/2√3

Solution:

(i) 2√3, 3/√2, -√7, √15

Let us simplify each fraction

$2\sqrt{3} = \sqrt{(4\times3)} = \sqrt{12}$

$3/\sqrt{2} = (3\times\sqrt{2})/(\sqrt{2}\times\sqrt{2}) = 3\sqrt{2}/2 = \sqrt{((9/4)\times2)} = \sqrt{(9/2)} = \sqrt{4.5}$

$-\sqrt{7} = -\sqrt{7}$

$\sqrt{15} = \sqrt{15}$

So,

The greatest real number = √15

Smallest real number = -√7

(ii) -3√2, 9/√5, -4, 4/3 √5, 3/2√3

Let us simplify each fraction

-3√2 = -√(9×2) = -√18

9/√5 = (9×√5)/(√5×√5) = 9√5/5 = √((81/25)×5) = √(81/5) = √16.2

-4 = -√16

4/3 √5 = √((16/9)×5) = √(80/9) = √8.88 = √8.8

3/2√3 = √((9/4)×3) = √(27/4) = √6.25

So,

The greatest real number = 9√5

Smallest real number = -3√2

10. Write in ascending order.

(i) 3√2, 2√3, √15, 4

(ii) 3√2, 2√8, 4, √50, 4√3

Solution:

(i) 3√2, 2√3, √15, 4

3√2 = √(9×2) =√18

2√3 = √(4×3) =√12

√15 = √15

4 = √16

Now, let us arrange in ascending order

√12, √15, √16, √18

So,

2√3, √15, 4, 3√2

(ii) 3√2, 2√8, 4, √50, 4√3

3√2 = √(9×2) =√18

2√8 = √(4×8) =√32

4 = √16

√50 = √50

4√3 =√(16×3) = √48

Now, let us arrange in ascending order

$\sqrt{16}, \sqrt{18}, \sqrt{32}, \sqrt{48}, \sqrt{50}$

So,

$4, 3\sqrt{2}, 2\sqrt{8}, 4\sqrt{3}, \sqrt{50}$

11. Write in descending order.

(i) $9/\sqrt{2}, 3/2\ \sqrt{5}, 4\sqrt{3}, 3\sqrt{(6/5)}$

(ii) $5/\sqrt{3}, 7/3\ \sqrt{2}, -\sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$

Solution:

(i) $9/\sqrt{2}, 3/2\ \sqrt{5}, 4\sqrt{3}, 3\sqrt{(6/5)}$

$9/\sqrt{2} = (9\times\sqrt{2})/(\sqrt{2}\times\sqrt{2}) = 9\sqrt{2}/2 = \sqrt{((81/4)\times2)} = \sqrt{(81/2)} = \sqrt{40.5}$

$3/2\ \sqrt{5} = \sqrt{((9/4)\times5)} = \sqrt{(45/4)} = \sqrt{11.25}$

$4\sqrt{3} = \sqrt{(16\times3)} = \sqrt{48}$

$3\sqrt{(6/5)} = \sqrt{((9\times6)/5)} = \sqrt{(54/5)} = \sqrt{10.8}$

Now, let us arrange in descending order

$\sqrt{48}, \sqrt{40.5}, \sqrt{11.25}, \sqrt{10.8}$

So,

$4\sqrt{3}, 9/\sqrt{2}, 3/2\ \sqrt{5}, 3\sqrt{(6/5)}$

(ii) $5/\sqrt{3}, 7/3\ \sqrt{2}, -\sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$

$5/\sqrt{3} = \sqrt{(25/3)} = \sqrt{8.33}$

$7/3\ \sqrt{2} = \sqrt{((49/9)\times2)} = \sqrt{98/9} = \sqrt{10.88}$

$-\sqrt{3} = -\sqrt{3}$

$3\sqrt{5} = \sqrt{(9\times5)} = \sqrt{45}$

$2\sqrt{7} = \sqrt{(4\times7)} = \sqrt{28}$

Now, let us arrange in descending order

$\sqrt{45}, \sqrt{28}, \sqrt{10.88..}, \sqrt{8.33..}, -\sqrt{3}$

So,

$3\sqrt{5}, 2\sqrt{7}, 7/3\sqrt{2}, 5/\sqrt{3}, -\sqrt{3}$

12. Arrange in ascending order.

$\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$

Solution:

Here we can express the given expressions as:

$\sqrt[3]{2} = 2^{1/3}$

$\sqrt{3} = 3^{1/2}$

$\sqrt[6]{5} = 5^{1/6}$

Let us make the roots common so,

$2^{1/3} = 2^{(2\times 1/2 \times 1/3)} = 4^{1/6}$

$3^{1/2} = 3^{(3\times 1/3 \times 1/2)} = 27^{1/6}$

$5^{1/6} = 5^{1/6}$

Now, let us arrange in ascending order,

$4^{1/6}, 5^{1/6}, 27^{1/6}$

So,

$2^{1/3}, 5^{1/6}, 3^{1/2}$

So,

$\sqrt[3]{2}, \sqrt[6]{5}, \sqrt{3}$

EXERCISE 1.5

1. Rationalize the denominator of the following:

(i) 3/4√5

(ii) 5√7 / √3

(iii) 3/(4 – √7)

(iv) 17/(3√2 + 1)

(v) 16/ (√41 – 5)

(vi) 1/ (√7 – √6)

(vii) 1/ (√5 + √2)

(viii) (√2 + √3) / (√2 – √3)

Solution:

(i) 3/4√5

Let us rationalize,

3/4√5 = (3×√5) /(4√5×√5)

= (3√5) / (4×5)

= (3√5) / 20

(ii) 5√7 / √3

Let us rationalize,

5√7 / √3 = (5√7×√3) / (√3×√3)

$= 5\sqrt{21}/3$

(iii) $3/(4 - \sqrt{7})$

Let us rationalize,

$3/(4 - \sqrt{7}) = [3\times(4 + \sqrt{7})] / [(4 - \sqrt{7}) \times (4 + \sqrt{7})]$

$= 3(4 + \sqrt{7}) / [4^2 - (\sqrt{7})^2]$

$= 3(4 + \sqrt{7}) / [16 - 7]$

$= 3(4 + \sqrt{7}) / 9$

$= (4 + \sqrt{7}) / 3$

(iv) $17/(3\sqrt{2} + 1)$

Let us rationalize,

$17/(3\sqrt{2} + 1) = 17(3\sqrt{2} - 1) / [(3\sqrt{2} + 1) (3\sqrt{2} - 1)]$

$= 17(3\sqrt{2} - 1) / [(3\sqrt{2})^2 - 1^2]$

$= 17(3\sqrt{2} - 1) / [9.2 - 1]$

$= 17(3\sqrt{2} - 1) / [18 - 1]$

$= 17(3\sqrt{2} - 1) / 17$

$= (3\sqrt{2} - 1)$

(v) $16/ (\sqrt{41} - 5)$

Let us rationalize,

$16/ (\sqrt{41} - 5) = 16(\sqrt{41} + 5) / [(\sqrt{41} - 5) (\sqrt{41} + 5)]$

$= 16(\sqrt{41} + 5) / [(\sqrt{41})^2 - 5^2]$

$= 16(\sqrt{41} + 5) / [41 - 25]$

$= 16(\sqrt{41} + 5) / [16]$

$= (\sqrt{41} + 5)$

(vi) $1/ (\sqrt{7} - \sqrt{6})$

Let us rationalize,

$1/ (\sqrt{7} - \sqrt{6}) = 1(\sqrt{7} + \sqrt{6}) / [(\sqrt{7} - \sqrt{6}) (\sqrt{7} + \sqrt{6})]$

$= (\sqrt{7} + \sqrt{6}) / [(\sqrt{7})^2 - (\sqrt{6})^2]$

$= (\sqrt{7} + \sqrt{6}) / [7 - 6]$

$= (\sqrt{7} + \sqrt{6}) / 1$

$= (\sqrt{7} + \sqrt{6})$

(vii) $1/ (\sqrt{5} + \sqrt{2})$

Let us rationalize,

1/ (√5 + √2) = 1(√5 – √2) / [(√5 + √2) (√5 – √2)]

= (√5 – √2) / [$(\sqrt{5})^2$ – $(\sqrt{2})^2$]

= (√5 – √2) / [5 – 2]

= (√5 – √2) / [3]

= (√5 – √2) /3

(viii) (√2 + √3) / (√2 – √3)

Let us rationalize,

(√2 + √3) / (√2 – √3) = [(√2 + √3) (√2 + √3)] / [(√2 – √3) (√2 + √3)]

= [$(\sqrt{2} + \sqrt{3})^2$] / [$(\sqrt{2})^2$ – $(\sqrt{3})^2$]

= [2 + 3 + 2√2√3] / [2 – 3]

= [5 + 2√6] / -1

= – (5 + 2√6)

2. Simplify each of the following by rationalizing the denominator:

(i) (7 + 3√5) / (7 – 3√5)

(ii) (3 – 2√2) / (3 + 2√2)

(iii) (5 – 3√14) / (7 + 2√14)

Solution:

(i) (7 + 3√5) / (7 – 3√5)

Let us rationalize the denominator, we get

(7 + 3√5) / (7 – 3√5) = [(7 + 3√5) (7 + 3√5)] / [(7 – 3√5) (7 + 3√5)]

= [$(7 + 3\sqrt{5})^2$] / [7^2 – $(3\sqrt{5})^2$]

= [7^2 + $(3\sqrt{5})^2$ + 2.7. 3√5] / [49 – 9.5]

= [49 + 9.5 + 42√5] / [49 – 45]

= [49 + 45 + 42√5] / [4]

= [94 + 42√5] / 4

= 2[47 + 21√5]/4

= [47 + 21√5]/2

(ii) (3 – 2√2) / (3 + 2√2)

Let us rationalize the denominator, we get

(3 – 2√2) / (3 + 2√2) = [(3 – 2√2) (3 – 2√2)] / [(3 + 2√2) (3 – 2√2)]

$= [(3 - 2\sqrt{2})^2] / [3^2 - (2\sqrt{2})^2]$

$= [3^2 + (2\sqrt{2})^2 - 2.3.2\sqrt{2}] / [9 - 4.2]$

$= [9 + 4.2 - 12\sqrt{2}] / [9 - 8]$

$= [9 + 8 - 12\sqrt{2}] / 1$

$= 17 - 12\sqrt{2}$

(iii) $(5 - 3\sqrt{14}) / (7 + 2\sqrt{14})$

Let us rationalize the denominator, we get

$(5 - 3\sqrt{14}) / (7 + 2\sqrt{14}) = [(5 - 3\sqrt{14})(7 - 2\sqrt{14})] / [(7 + 2\sqrt{14})(7 - 2\sqrt{14})]$

$= [5(7 - 2\sqrt{14}) - 3\sqrt{14}(7 - 2\sqrt{14})] / [7^2 - (2\sqrt{14})^2]$

$= [35 - 10\sqrt{14} - 21\sqrt{14} + 6.14] / [49 - 4.14]$

$= [35 - 31\sqrt{14} + 84] / [49 - 56]$

$= [119 - 31\sqrt{14}] / [-7]$

$= -[119 - 31\sqrt{14}] / 7$

$= [31\sqrt{14} - 119] / 7$

3. Simplify:

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})] - [2\sqrt{5} / (\sqrt{6} + \sqrt{5})] - [3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

Solution:

Let us simplify individually,

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})]$

Let us rationalize the denominator,

$7\sqrt{3} / (\sqrt{10} + \sqrt{3}) = [7\sqrt{3}(\sqrt{10} - \sqrt{3})] / [(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3})]$

$= [7\sqrt{3}.\sqrt{10} - 7\sqrt{3}.\sqrt{3}] / [(\sqrt{10})^2 - (\sqrt{3})^2]$

$= [7\sqrt{30} - 7.3] / [10 - 3]$

$= 7[\sqrt{30} - 3] / 7$

$= \sqrt{30} - 3$

Now,

$[2\sqrt{5} / (\sqrt{6} + \sqrt{5})]$

Let us rationalize the denominator, we get

$2\sqrt{5} / (\sqrt{6} + \sqrt{5}) = [2\sqrt{5}(\sqrt{6} - \sqrt{5})] / [(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})]$

$= [2\sqrt{5}.\sqrt{6} - 2\sqrt{5}.\sqrt{5}] / [(\sqrt{6})^2 - (\sqrt{5})^2]$

$= [2\sqrt{30} - 2.5] / [6 - 5]$

$= [2\sqrt{30} - 10] / 1$

$= 2\sqrt{30} - 10$

Now,

$[3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

Let us rationalize the denominator, we get

$3\sqrt{2} / (\sqrt{15} + 3\sqrt{2}) = [3\sqrt{2} (\sqrt{15} - 3\sqrt{2})] / [(\sqrt{15} + 3\sqrt{2}) (\sqrt{15} - 3\sqrt{2})]$

$= [3\sqrt{2}.\sqrt{15} - 3\sqrt{2}.3\sqrt{2}] / [(\sqrt{15})^2 - (3\sqrt{2})^2]$

$= [3\sqrt{30} - 9.2] / [15 - 9.2]$

$= [3\sqrt{30} - 18] / [15 - 18]$

$= 3[\sqrt{30} - 6] / [-3]$

$= [\sqrt{30} - 6] / -1$

$= 6 - \sqrt{30}$

So, according to the question let us substitute the obtained values,

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})] - [2\sqrt{5} / (\sqrt{6} + \sqrt{5})] - [3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

$= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (6 - \sqrt{30})$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 - 6 + \sqrt{30}$

$= 2\sqrt{30} - 2\sqrt{30} - 3 + 10 - 6$

$= 1$

4. Simplify:

$[1/(\sqrt{4} + \sqrt{5})] + [1/(\sqrt{5} + \sqrt{6})] + [1/(\sqrt{6} + \sqrt{7})] + [1/(\sqrt{7} + \sqrt{8})] + [1/(\sqrt{8} + \sqrt{9})]$

Solution:

Let us simplify individually,

$[1/(\sqrt{4} + \sqrt{5})]$

Rationalize the denominator, we get

$[1/(\sqrt{4} + \sqrt{5})] = [1(\sqrt{4} - \sqrt{5})] / [(\sqrt{4} + \sqrt{5}) (\sqrt{4} - \sqrt{5})]$

$= [(\sqrt{4} - \sqrt{5})] / [(\sqrt{4})^2 - (\sqrt{5})^2]$

$= [(\sqrt{4} - \sqrt{5})] / [4 - 5]$

$= [(\sqrt{4} - \sqrt{5})] / -1$

$= -(\sqrt{4} - \sqrt{5})$

Now,

$[1/(\sqrt{5} + \sqrt{6})]$

Rationalize the denominator, we get

$[1/(\sqrt{5} + \sqrt{6})] = [1(\sqrt{5} - \sqrt{6})] / [(\sqrt{5} + \sqrt{6}) (\sqrt{5} - \sqrt{6})]$

$= [(\sqrt{5} - \sqrt{6})] / [(\sqrt{5})^2 - (\sqrt{6})^2]$

$= [(\sqrt{5} - \sqrt{6})] / [5 - 6]$

= [(√5 – √6)] / -1

= -(√5 – √6)

Now,

[1/(√6 + √7)]
Rationalize the denominator, we get

[1/(√6 + √7)] = [1(√6 – √7)] / [(√6 + √7) (√6 – √7)]
= [(√6 – √7)] / [(√6)^2 – (√7)^2]

= [(√6 – √7)] / [6 – 7]

= [(√6 – √7)] / -1

= -(√6 – √7)

Now,

[1/(√7 + √8)]
Rationalize the denominator, we get

[1/(√7 + √8)] = [1(√7 – √8)] / [(√7 + √8) (√7 – √8)]
= [(√7 – √8)] / [(√7)^2 – (√8)^2]

= [(√7 – √8)] / [7 – 8]

= [(√7 – √8)] / -1

= -(√7 – √8)

Now,

[1/(√8 + √9)]
Rationalize the denominator, we get

[1/(√8 + √9)] = [1(√8 – √9)] / [(√8 + √9) (√8 – √9)]
= [(√8 – √9)] / [(√8)^2 – (√9)^2]

= [(√8 – √9)] / [8 – 9]

= [(√8 – √9)] / -1

= -(√8 – √9)

So, according to the question let us substitute the obtained values,

[1/(√4 + √5)] + [1/(√5 + √6)] + [1/(√6 + √7)] + [1/(√7 + √8)] + [1/(√8 + √9)]
= -(√4 – √5) + -(√5 – √6) + -(√6 – √7) + -(√7 – √8) + -(√8 – √9)

= -√4 + √5 – √5 + √6 – √6 + √7 – √7 + √8 – √8 + √9

= -√4 + √9

= -2 + 3

= 1

5. Give a and b are rational numbers. Find a and b if:

(i) $[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = -19/11 + a\sqrt{5}$

(ii) $[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = a - b\sqrt{6}$

(iii) $\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\} - \{[7 - \sqrt{5}]/[7 + \sqrt{5}]\} = a + 7/11\ b\sqrt{5}$

Solution:

(i) $[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = -19/11 + a\sqrt{5}$

Let us consider LHS

$[3 - \sqrt{5}] / [3 + 2\sqrt{5}]$
Rationalize the denominator,

$[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = [(3 - \sqrt{5})(3 - 2\sqrt{5})] / [(3 + 2\sqrt{5})(3 - 2\sqrt{5})]$
$= [3(3 - 2\sqrt{5}) - \sqrt{5}(3 - 2\sqrt{5})] / [3^2 - (2\sqrt{5})^2]$

$= [9 - 6\sqrt{5} - 3\sqrt{5} + 2.5] / [9 - 4.5]$

$= [9 - 6\sqrt{5} - 3\sqrt{5} + 10] / [9 - 20]$

$= [19 - 9\sqrt{5}] / -11$

$= -19/11 + 9\sqrt{5}/11$

So when comparing with RHS

$-19/11 + 9\sqrt{5}/11 = -19/11 + a\sqrt{5}$

Hence, value of $a = 9/11$

(ii) $[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = a - b\sqrt{6}$

Let us consider LHS

$[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}]$
Rationalize the denominator,

$[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = [(\sqrt{2} + \sqrt{3})(3\sqrt{2} + 2\sqrt{3})] / [(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 2\sqrt{3})]$
$= [\sqrt{2}(3\sqrt{2} + 2\sqrt{3}) + \sqrt{3}(3\sqrt{2} + 2\sqrt{3})] / [(3\sqrt{2})^2 - (2\sqrt{3})^2]$

$= [3.2 + 2\sqrt{2}\sqrt{3} + 3\sqrt{2}\sqrt{3} + 2.3] / [9.2 - 4.3]$

$= [6 + 2\sqrt{6} + 3\sqrt{6} + 6] / [18 - 12]$

$= [12 + 5\sqrt{6}] / 6$

$= 12/6 + 5\sqrt{6}/6$

$= 2 + 5\sqrt{6}/6$

$= 2 - (-5\sqrt{6}/6)$

So when comparing with RHS

$2 - (-5\sqrt{6}/6) = a - b\sqrt{6}$

Hence, value of $a = 2$ and $b = -5/6$

(iii) $\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\} - \{[7 - \sqrt{5}]/[7 + \sqrt{5}]\} = a + 7/11\ b\sqrt{5}$

Let us consider LHS

Since there are two terms, let us solve individually

{[7 + √5]/[7 – √5]}

Rationalize the denominator,

[7 + √5]/[7 – √5] = [(7 + √5) (7 + √5)] / [(7 – √5) (7 + √5)]
= [$(7 + √5)^2$] / [7^2 – $(√5)^2$]

= [7^2 + $(√5)^2$ + 2.7.√5] / [49 – 5]

= [49 + 5 + 14√5] / [44]

= [54 + 14√5] / 44

Now,

{[7 – √5]/[7 + √5]}

Rationalize the denominator,

[7 – √5]/[7 + √5] = (7 – √5) (7 – √5)] / [(7 + √5) (7 – √5)]
= [$(7 – √5)^2$] / [7^2 – $(√5)^2$]

= [7^2 + $(√5)^2$ – 2.7.√5] / [49 – 5]

= [49 + 5 – 14√5] / [44]

= [54 – 14√5] / 44

So, according to the question

{[7 + √5]/[7 – √5]} – {[7 – √5]/[7 + √5]}

By substituting the obtained values,

= {[54 + 14√5] / 44} – {[54 – 14√5] / 44}

= [54 + 14√5 – 54 + 14√5]/44

= 28√5/44

= 7√5/11

So when comparing with RHS

7√5/11 = a + 7/11 b√5

Hence, value of a = 0 and b = 1

6. If {[7 + 3√5] / [3 + √5]} – {[7 – 3√5] / [3 – √5]} = p + q√5, find the value of p and q where p and q are rational numbers.

Solution:

Let us consider LHS

Since there are two terms, let us solve individually

{[7 + 3√5] / [3 + √5]}

Rationalize the denominator,

[7 + 3√5] / [3 + √5] = [(7 + 3√5) (3 – √5)] / [(3 + √5) (3 – √5)]
= [7(3 – √5) + 3√5(3 – √5)] / [3^2 – $(\sqrt{5})^2$]

= [21 – 7√5 + 9√5 – 3.5] / [9 – 5]

= [21 + 2√5 – 15] / [4]

= [6 + 2√5] / 4

= 2[3 + √5]/4

= [3 + √5] /2

Now,

{[7 – 3√5] / [3 – √5]}

Rationalize the denominator,

[7 – 3√5] / [3 – √5] = [(7 – 3√5) (3 + √5)] / [(3 – √5) (3 + √5)]
= [7(3 + √5) – 3√5(3 + √5)] / [3^2 – $(\sqrt{5})^2$]

= [21 + 7√5 – 9√5 – 3.5] / [9 – 5]

= [21 – 2√5 – 15] / 4

= [6 – 2√5]/4

= 2[3 – √5]/4

= [3 – √5]/2

So, according to the question

{[7 + 3√5] / [3 + √5]} – {[7 – 3√5] / [3 – √5]}

By substituting the obtained values,

= {[3 + √5] /2} – {[3 – √5] /2}

= [3 + √5 – 3 + √5]/2

= [2√5]/2

= √5

So when comparing with RHS

√5 = p + q√5

Hence, value of p = 0 and q = 1

7. Rationalise the denominator of the following and hence evaluate by taking √2 = 1.414, √3 = 1.732, upto three places of decimal:

(i) √2/(2 + √2)

(ii) 1/(√3 + √2)

Solution:

(i) √2/(2 + √2)

By rationalizing the denominator,

√2/(2 + √2) = [√2(2 – √2)] / [(2 + √2) (2 – √2)]

= [2√2 – 2] / [2^2 – $(\sqrt{2})^2$]

= [2√2 – 2] / [4 – 2]

= 2[√2 – 1] / 2

= √2 – 1

= 1.414 – 1

= 0.414

(ii) 1/(√3 + √2)

By rationalizing the denominator,

1/(√3 + √2) = [1(√3 – √2)] / [(√3 + √2) (√3 – √2)]

= [(√3 – √2)] / [$(\sqrt{3})^2$ – $(\sqrt{2})^2$]

= [(√3 – √2)] / [3 – 2]

= [(√3 – √2)] / 1

= (√3 – √2)

= 1.732 – 1.414

= 0.318

8. If a = 2 + √3, find 1/a, (a – 1/a)

Solution:

Given:

a = 2 + √3

So,

1/a = 1/ (2 + √3)

By rationalizing the denominator,

1/ (2 + √3) = [1(2 – √3)] / [(2 + √3) (2 – √3)]

= [(2 – √3)] / [2^2 – $(\sqrt{3})^2$]

= [(2 – √3)] / [4 – 3]

= (2 – √3)

Then,

a – 1/a = 2 + √3 – (2 – √3)

= 2 + √3 – 2 + √3

= 2√3

9. Solve:

If x = 1 – √2, find 1/x, $(x – 1/x)^4$

Solution:

Given:

x = 1 – √2

so,

1/x = 1/(1 – √2)

By rationalizing the denominator,

1/ (1 – √2) = [1(1 + √2)] / [(1 – √2) (1 + √2)]

= [(1 + √2)] / [1^2 – $(\sqrt{2})^2$]

= [(1 + √2)] / [1 – 2]

= (1 + √2) / -1

= -(1 + √2)

Then,

$(x – 1/x)^4$ = $[1 – \sqrt{2} – (-1 – \sqrt{2})]^4$

= $[1 – \sqrt{2} + 1 + \sqrt{2}]^4$

= 2^4

= 16

10. Solve:

If x = 5 – 2√6, find 1/x, (x^2 – 1/x^2)

Solution:

Given:

x = 5 – 2√6

so,

1/x = 1/(5 – 2√6)

By rationalizing the denominator,

1/(5 – 2√6) = [1(5 + 2√6)] / [(5 – 2√6) (5 + 2√6)]

$= [(5 + 2\sqrt{6})] / [5^2 – (2\sqrt{6})^2]$

$= [(5 + 2\sqrt{6})] / [25 – 4.6]$

$= [(5 + 2\sqrt{6})] / [25 – 24]$

$= (5 + 2\sqrt{6})$

Then,

$x + 1/x = 5 – 2\sqrt{6} + (5 + 2\sqrt{6})$

$= 10$

Square on both sides we get

$(x + 1/x)^2 = 10^2$

$x^2 + 1/x^2 + 2x.1/x = 100$

$x^2 + 1/x^2 + 2 = 100$

$x^2 + 1/x^2 = 100 – 2$

$= 98$

11. If p = (2-√5)/(2+√5) and q = (2+√5)/(2-√5), find the values of

(i) p + q

(ii) p – q

(iii) $p^2 + q^2$

(iv) $p^2 – q^2$

Solution:

Given:

$p = (2-\sqrt{5})/(2+\sqrt{5})$ and $q = (2+\sqrt{5})/(2-\sqrt{5})$

(i) p + q

$[(2-\sqrt{5})/(2+\sqrt{5})] + [(2+\sqrt{5})/(2-\sqrt{5})]$

So by rationalizing the denominator, we get

$= [(2 – \sqrt{5})^2 + (2 + \sqrt{5})^2] / [2^2 – (\sqrt{5})^2]$

$= [4 + 5 – 4\sqrt{5} + 4 + 5 + 4\sqrt{5}] / [4 – 5]$

$= [18]/-1$

$= -18$

(ii) p – q

$[(2-\sqrt{5})/(2+\sqrt{5})] – [(2+\sqrt{5})/(2-\sqrt{5})]$

So by rationalizing the denominator, we get

$= [(2 – \sqrt{5})^2 – (2 + \sqrt{5})^2] / [2^2 – (\sqrt{5})^2]$

$= [4 + 5 – 4\sqrt{5} – (4 + 5 + 4\sqrt{5})] / [4 – 5]$

$= [9 – 4\sqrt{5} – 9 – 4\sqrt{5}] / -1$

$= [-8\sqrt{5}]/-1$

$= 8\sqrt{5}$

(iii) $p^2 + q^2$

We know that $(p + q)^2 = p^2 + q^2 + 2pq$

So,

$p^2 + q^2 = (p + q)^2 – 2pq$

$pq = [(2-\sqrt{5})/(2+\sqrt{5})] \times [(2+\sqrt{5})/(2-\sqrt{5})]$

$= 1$

$p + q = -18$

so,

$p^2 + q^2 = (p + q)^2 – 2pq$

$= (-18)^2 – 2(1)$

$= 324 – 2$

$= 322$

(iv) $p^2 – q^2$

We know that, $p^2 – q^2 = (p + q)(p – q)$

So, by substituting the values

$p^2 – q^2 = (p + q)(p – q)$

$= (-18)(8\sqrt{5})$

$= -144\sqrt{5}$

12. If $x = (\sqrt{2} – 1)/(\sqrt{2} + 1)$ and $y = (\sqrt{2} + 1)/(\sqrt{2} – 1)$, find the value of $x^2 + 5xy + y^2$.

Solution:

Given:

$x = (\sqrt{2} – 1)/(\sqrt{2} + 1)$ and $y = (\sqrt{2} + 1)/(\sqrt{2} – 1)$

$x + y = [(\sqrt{2} – 1)/(\sqrt{2} + 1)] + [(\sqrt{2} + 1)/(\sqrt{2} – 1)]$

By rationalizing the denominator,

$= [(\sqrt{2} – 1)^2 + (\sqrt{2} + 1)^2] / [(\sqrt{2})^2 – 1^2]$

$= [2 + 1 – 2\sqrt{2} + 2 + 1 + 2\sqrt{2}] / [2 – 1]$

$= [6] / 1$

= 6

xy = [(√2 – 1)/(√2 + 1)] × [(√2 + 1)/(√2 – 1)]

= 1

We know that

$x^2 + 5xy + y^2 = x^2 + y^2 + 2xy + 3xy$

It can be written as

$= (x + y)^2 + 3xy$

Substituting the values

$= 6^2 + 3 \times 1$

So we get

= 36 + 3

= 39

Chapter test

1. Without actual division, find whether the following rational numbers are terminating decimals or recurring decimals:

(i) 13/45

(ii) -5/56

(iii) 7/125

(iv) -23/80

(v) – 15/66

In case of terminating decimals, write their decimal expansions.

Solution:

(i) We know that

The fraction whose denominator is the multiple of 2 or 5 or both is a terminating decimal

In 13/45

45 = 3 × 3 × 5

Hence, it is not a terminating decimal.

(ii) In -5/56

56 = 2 × 2 × 2 × 7

Hence, it is not a terminating decimal.

(iii) In 7/125

125 = 5 × 5 × 5

We know that

$$\frac{7}{125} = \frac{7 \times 8}{125 \times 8} = \frac{56}{1000} = 0.056$$

Hence, it is a terminating decimal.

(iv) In -23/80

80 = 2 × 2 × 2 × 2 × 5

We know that

$$\frac{-23}{80} = \frac{-23 \times 125}{80 \times 125} = \frac{-2875}{10000} = -0.2875$$

Hence, it is a terminating decimal.

(v) In – 15/66

66 = 2 × 3 × 11

Hence, it is not a terminating decimal.

2. Express the following recurring decimals as vulgar fractions:

$$\mathbf{(i) 1.\overline{345}}$$
$$\mathbf{(ii) 2.\overline{357}}$$

Solution:

(i) We know that

$$x = 1.\overline{345} = 1.34545...(1)$$

Now multiply both sides of equation (1) by 10

10x = 13.4545 ….. (2)

Again multiply both sides of equation (2) by 100

1000x = 1345.4545 …… (3)

By subtracting equation (2) from (3)

990x = 1332

By further calculation

x = 1332/990 = 74/55

(ii) We know that

$$x = 2.\overline{357} = 2.357357...(1)$$

Now multiply both sides of equation (1) by 1000

1000x = 2357.357357 ….. (2)

By subtracting equation (1) from (2)

999x = 2355

By further calculation

x = 2355/999

3. Insert a rational number between 5/9 and 7/13, and arrange in ascending order.

Solution:

We know that

A rational number between 5/9 and 7/13

$$\frac{\frac{5}{9}+\frac{7}{13}}{2}=\frac{\frac{65+63}{117}}{2}$$

$$By\ further\ caculation$$

$$=\frac{128}{117\times 2}$$

$$=\frac{64}{117}$$

$$Here$$

$$\frac{7}{13}<\frac{64}{117}<\frac{5}{9}$$

Therefore, in ascending order – 7/13, 64/117, 5/9.

4. Insert four rational numbers between 4/5 and 5/6.

Solution:

We know that

Rational numbers between 4/5 and 5/6

Here LCM of 5, 6 = 30

$$\frac{4}{5}=\frac{4\times 6}{5\times 6}=\frac{24}{30}=\frac{48}{60}=\frac{96}{120}=\frac{192}{240}$$

$$\frac{5}{6}=\frac{5\times 5}{6\times 5}=\frac{25}{30}=\frac{50}{60}=\frac{100}{120}=\frac{200}{240}$$

So the four rational numbers are

121/150, 122/150, 123/150, 124/150

By further simplification

121/150, 61/75, 41/50, 62/75

5. Prove that the reciprocal of an irrational number is irrational.

Solution:

Consider x as an irrational number

Reciprocal of x is 1/x

If 1/x is a non-zero rational number

Then x × 1/x will also be an irrational number.

We know that the product of a non-zero rational number and irrational number is also irrational.

If x × 1/x = 1 is rational number

Our assumption is wrong

So 1/x is also an irrational number.

Therefore, the reciprocal of an irrational number is also an irrational number.

6. Prove that the following numbers are irrational:

(i) $\sqrt{8}$
(ii) $\sqrt{14}$
(iii) $\sqrt[3]{2}$

Solution:

(i) √8

If √8 is a rational number

Consider √8 = p/q where p and q are integers

q > 0 and p and q have no common factor

By squaring on both sides

$8 = p^2/q^2$

So we get

$p^2 = 8q^2$

We know that

$8p^2$ is divisible by 8

p^2 is also divisible by 8

p is divisible by 8

Consider p = 8k where k is an integer

By squaring on both sides

$p^2 = (8k)^2$

$p^2 = 64k^2$

We know that

$64k^2$ is divisible by 8

p^2 is divisible by 8

p is divisible by 8

Here p and q both are divisible by 8

So our supposition is wrong

Therefore, √8 is an irrational number.

(ii) √14

If √14 is a rational number

Consider √14 = p/q where p and q are integers

q ≠ 0 and p and q have no common factor

By squaring on both sides

$14 = p^2/q^2$

So we get

$p^2 = 14q^2$ ….. (1)

We know that

p^2 is also divisible by 2

p is divisible by 2

Consider p = 2m

Substitute the value of p in equation (1)

$(2m)^2 = 13q^2$

So we get

$4m^2 = 14q^2$

$2m^2 = 7q^2$

We know that

q^2 is divisible by 2

q is divisible by 2

Here p and q have 2 as the common factor which is not possible

Therefore, √14 is an irrational number.

(iii) $\sqrt[3]{2}$

If $\sqrt[3]{2}$ is a rational number

Consider $\sqrt[3]{2}$ = p/q where p and q are integers

q > 0 and p and q have no common factor

By cubing on both sides

$2 = p^3/q^3$

So we get

$p^3 = 2q^3$ ….. (1)

We know that

$2q^3$ is also divisible by 2

p^3 is divisible by 2

p is divisible by 2

Consider p = 2k where k is an integer

By cubing both sides

$p^3 = (2k)^3$

$p^3 = 8k^3$

So we get

$2q^3 = 8k^3$

$q^3 = 4k^3$

We know that

$4k^3$ is divisible by 2

q^3 is divisible by 2

q is divisible by 2

Here p and q are divisible by 2

So our supposition is wrong

Therefore, $\sqrt[3]{2}$ is an irrational number.

7. Prove that √3 is a rational number. Hence show that 5 – √3 is an irrational number.

Solution:

If √3 is a rational number

Consider √3 = p/q where p and q are integers

q > 0 and p and q have no common factor

By squaring both sides

$3 = p^2/q^2$

So we get

$P^2 = 3q^2$

We know that

$3q^2$ is divisible by 3

p^2 is divisible by 3

p is divisible by 3

Consider p = 3 where k is an integer

By squaring on both sides

$P^2 = 9k^2$

$9k^2$ is divisible by 3

p^2 is divisible by 3

$3q^2$ is divisible by 3

q^2 is divisible by 3

q is divisible by 3

Here p and q are divisible by 3

So our supposition is wrong

Therefore, √3 is an irrational number.

In 5 – √3

5 is a rational number

√3 is an irrational number (proved)

We know that

Difference of a rational number and irrational number is also an irrational number

So 5 – √3 is an irrational number.

Therefore, it is proved.

8. Prove that the following numbers are irrational:

(i) 3 + √5

(ii) 15 – 2√7

$(iii)\frac{1}{3-\sqrt{5}}$

Solution:

(i) If 3 + √5 is a rational number say x

Consider 3 + √5 = x

It can be written as

√5 = x – 3

Here x – 3 is a rational number

√5 is also a rational number.

Consider √5 = p/q where p and q are integers

q > 0 and p and q have no common factor

By squaring both sides

$5 = p^2/q^2$

$p^2 = 5q^2$

We know that

$5q^2$ is divisible by 5

p^2 is divisible by 5

p is divisible 5

Consider p = 5k where k is an integer

By squaring on both sides

$p^2 = 25k^2$

So we get

$5q^2 = 25k^2$

$q^2 = 5k^2$

Here

$5k^2$ is divisible by 5

q^2 is divisible by 5

q is divisible by 5

Here p and q are divisible by 5

So our supposition is wrong

√5 is an irrational number

$3 + \sqrt{5}$ is also an irrational number.

Therefore, it is proved.

(ii) If $15 – 2\sqrt{7}$ is a rational number say x

Consider $15 – 2\sqrt{7} = x$

It can be written as

$2\sqrt{7} = 15 – x$

So we get

$\sqrt{7} = (15 – x)/ 2$

Here

$(15 – x)/ 2$ is a rational number

$\sqrt{7}$ is a rational number

Consider $\sqrt{7} = p/q$ where p and q are integers

$q > 0$ and p and q have no common factor

By squaring on both sides

$7 = p^2/q^2$

$p^2 = 7q^2$

Here

$7q^2$ is divisible by 7

p^2 is divisible by 7

p is divisible by 7

Consider $p = 7k$ where k is an integer

By squaring on both sides

$p^2 = 49k^2$

It can be written as

$7q^2 = 49k^2$

$q^2 = 7k^2$

Here

$7k^2$ is divisible by 7

q^2 is divisible by 7

q is divisible by 7

Here p and q are divisible by 7

So our supposition is wrong

√7 is an irrational number

15 – 2√7 is also an irrational number.

Therefore, it is proved.

$$\textbf{(iii)}\frac{1}{3-\sqrt{5}}$$

By rationalizing the denominator

$$\frac{1}{3-\sqrt{5}} = \frac{1\times(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$By\ further\ calculation$$

$$= \frac{3+\sqrt{5}}{9-5}$$

$$= \frac{3+\sqrt{5}}{4}$$

$$So\ we\ get$$

$$= \frac{3}{4} + \frac{\sqrt{5}}{4}$$

Here

¾ is a rational number and √5/4 is an irrational number

We know that

Sum of a rational and an irrational number is an irrational number.

Therefore, it is proved.

9. Rationalise the denominator of the following:

$$\textbf{(i)}\frac{10}{2\sqrt{2}+\sqrt{3}}$$

$$\textbf{(ii)}\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$\textbf{(iii)}\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

Solution:

$$(i)\frac{10}{2\sqrt{2}+\sqrt{3}} = \frac{10}{2\sqrt{2}+\sqrt{3}} \times \frac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}}$$

$By\ further\ calculation$

$$= \frac{10(2\sqrt{2}-\sqrt{3})}{(2\sqrt{2})^2-(\sqrt{3})^2}$$

$It\ can\ be\ written\ as$

$$= \frac{10(2\sqrt{2}-\sqrt{3})}{8-3}$$

$$= \frac{10(2\sqrt{2}-\sqrt{3})}{5}$$

$So\ we\ get$

$$= 2(2\sqrt{2}-\sqrt{3})$$

$$(ii)\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}\times\frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$$

By further calculation

$$=\frac{7\sqrt{44}-7\sqrt{54}-5\sqrt{96}+5\sqrt{36}}{(\sqrt{48})^2-(\sqrt{18})^2}$$

It can be written as

$$=\frac{7\times 12-7\times 3\sqrt{6}-5\times 4\sqrt{6}+5\times 6}{48-18}$$

By further simplification

$$=\frac{84-21\sqrt{6}-20\sqrt{6}+30}{30}$$

So we get

$$=\frac{114-41\sqrt{6}}{30}$$
$$=\frac{114}{30}-\frac{41}{30}\sqrt{6}$$
$$=\frac{57}{15}-\frac{41}{30}\sqrt{6}$$

$$(iii)\frac{1}{\sqrt{3}-\sqrt{2}+1}=\frac{1}{\sqrt{3}-(\sqrt{2}-1)}\times\frac{\sqrt{3}+(\sqrt{2}-1)}{\sqrt{3}+(\sqrt{2}-1)}$$

It can be written as

$$=\frac{\sqrt{3}+\sqrt{2}-1}{(\sqrt{3})^2-(\sqrt{2}-1)^2}$$

By further calculation

$$=\frac{\sqrt{3}+\sqrt{2}-1}{3-(2+1-2\sqrt{2})}$$

So we get

$$= \frac{\sqrt{3}+\sqrt{2}-1}{3-3+2\sqrt{2}}$$

$Multiply\, and\, divide\, by\, \sqrt{2}$

$$= \frac{\sqrt{3}+\sqrt{2}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$We\, can\, write\, it\, as$

$$= \frac{\sqrt{6}+\sqrt{4}-\sqrt{2}}{2\times 2}$$

$$= \frac{2+\sqrt{6}-\sqrt{2}}{4}$$

10. If p, q are rational numbers and p – √15q = 2√3 – √5/4√3 – 3√5, find the values of p and q.

Solution:

It is given that

$$p-\sqrt{15}q = \frac{2\sqrt{3}-\sqrt{5}}{4\sqrt{3}-3\sqrt{5}}$$

$Rationalising\, the\, denominator$

$$= \frac{2\sqrt{3}-\sqrt{5}}{4\sqrt{3}-3\sqrt{5}} \times \frac{4\sqrt{3}+3\sqrt{5}}{4\sqrt{3}+3\sqrt{5}}$$

$By\, further\, calculation$

$$= \frac{8\times 3+6\sqrt{15}-4\sqrt{15}-3\times 5}{(4\sqrt{3})^2-(3\sqrt{5})^2}$$

$It\, can\, be\, written\, as$

$$= \frac{24+2\sqrt{15}-15}{48-45}$$

$So\, we\, get$

$$= \frac{9+2\sqrt{15}}{3}$$

$$Separating\ the\ terms$$

$$= \frac{9}{3} + \frac{2}{3}\sqrt{15}$$

$$We\ get$$

$$= 3 + \frac{2}{3}\sqrt{15}$$

By comparing both sides

p = 3 and q = -2/3

11. If x = 1/3 + 2√2, then find the value of x – 1/x.

Solution:

$$x = \frac{1}{3 + 2\sqrt{2}} = \frac{1(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}$$

$$By\ further\ calculation$$

$$= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$$

$$So\ we\ get$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8}$$

$$= 3 - 2\sqrt{2}$$

Here

1/x = 3 + 2√2/1 = √3 + 2√2

We know that

x – 1/x = (3 – 2√2) – (3 + 2√2)

By further calculation

= 3 – 2√2 – 3 – 2√2

So we get

= -4√2

$$12.(i) \text{If } x = \frac{7+3\sqrt{5}}{7-3\sqrt{5}}, \text{ find the value of } x^2 + \frac{1}{x^2}.$$

$$(ii) \text{If } x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \text{ and } y = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}, \text{ then find the value of } x^2 + xy + y^2.$$

$$(iii) \text{If } x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \text{ and } y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, \text{ find the value of } x^3 + y^3.$$

$$\text{Hint.}(iii) x^3 + y^3 = (x+y)^3 - 3xy(x+y).$$

Solution:

$$(i) x = \frac{7+3\sqrt{5}}{7-3\sqrt{5}} = \frac{(7+3\sqrt{5})(7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})}$$

By rationalising the denominator

$$= \frac{(7+3\sqrt{5})^2}{(7)^2-(3\sqrt{5})^2}$$

It can be written as

$$= \frac{49 \times 45 + 2 \times 7 \times 3\sqrt{5}}{49-45}$$

By further calculation

$$= \frac{94+42\sqrt{5}}{4}$$

Dividing by 2

$$= \frac{47+21\sqrt{5}}{2}$$

We know that

$$\frac{1}{x} = \frac{2}{47+21\sqrt{5}}$$

Rationalising the denominator

$$= \frac{2(47-21\sqrt{5})}{(47+21\sqrt{5})(47-21\sqrt{5})}$$

$$= \frac{2(47-21\sqrt{5})}{(47)^2-(21\sqrt{5})^2}$$

So we get

$$= \frac{2(47-21\sqrt{5})}{2209-2205}$$

$$= \frac{2(47-21\sqrt{5})}{4}$$

Dividing by 2

$$= \frac{47-21\sqrt{5}}{2}$$

Here

$$x+\frac{1}{x} = \frac{47+21\sqrt{5}}{2} + \frac{47-21\sqrt{5}}{2}$$

By further calculation

$$= \frac{47+21\sqrt{5}+47-21\sqrt{5}}{2}$$

$$= \frac{94}{2}$$

$$= 47$$

By squaring on both sides

$$(x+\frac{1}{x})^2 = 47^2$$

On further simplification

$$x^2+\frac{1}{x^2}+2 = 2209$$

So we get

$$x^2+\frac{1}{x^2} = 2209-2 = 2207$$

$$(ii) x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}, y = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$By\, rationalising\, the\, denominator$

$$x = \frac{(\sqrt{5}-\sqrt{2})(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$By\, further\, calculation$

$$= \frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$
$$= \frac{5+2-2\sqrt{5}\sqrt{2}}{5-2}$$
$$= \frac{7-2\sqrt{10}}{3}$$

$Here$

$$y = \frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$$

$By\, further\, calculation$

$$= \frac{5+2+2\sqrt{5}\sqrt{2}}{\sqrt{5})^2-(\sqrt{2})^2}$$

$So\, we\, get$

$$= \frac{7+2\sqrt{10}}{5-2}$$
$$= \frac{7+2\sqrt{10}}{3}$$

$We\, know\, that$

$$x^2+xy+y^2 = (\frac{7-2\sqrt{10}}{3})^2 + \frac{7-2\sqrt{10}}{3}\frac{7+2\sqrt{10}}{3} + (\frac{7+2\sqrt{10}}{3})^2$$

$By\, further\, calculation$

$$= \frac{7^2 + (2\sqrt{10})^2 - 2 \times 7 \times 2\sqrt{10}}{9} + \frac{7^2 - (2\sqrt{10})^2}{9} + \frac{7^2 + (2\sqrt{10})^2 + 2 \times 7 \times 2\sqrt{10}}{9}$$

It can be written as

$$= \frac{49 + 40 - 28\sqrt{10} + 49 - 40 + 49 + 40 + 28\sqrt{10}}{9}$$

So we get

$$= \frac{147 + 40}{9}$$

$$= \frac{187}{9}$$

$$= 20\frac{7}{9}$$

$(iii) x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, \ y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$By\, rationalising\ the\ denominators$

$$x = \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$It\ can\ be\ written\ as$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$By\ further\ calculation$

$$= \frac{3+2-2\sqrt{3}\sqrt{2}}{3-2}$$

$$= \frac{5-2\sqrt{6}}{1}$$

$$= 5-2\sqrt{6}$$

$Here$

$$y = \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$It\ can\ be\ written\ as$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$By\ further\ calculation$

$$= \frac{3+2+2\sqrt{3}\sqrt{2}}{3-2}$$

$So\ we\ get$

$$= \frac{5+2\sqrt{6}}{1}$$

$$= 5 + 2\sqrt{6}$$

$$We\, know\, that$$

$$x + y = 5 - 2\sqrt{6} + 5 + 2\sqrt{6} = 10$$

$$It\, can\, be\, written\, as$$

$$xy = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = 1$$

$$Here$$

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$Substituting\, the\, values$$

$$= 10^3 + 3 \times 1 \times 10$$

=1000 – 30

= 970

13. Write the following real numbers in descending order:

$$\sqrt{2}, 3.5, \sqrt{10}, -\frac{5}{\sqrt{2}}, \frac{5}{2}\sqrt{3}$$

Solution:

We know that

√2 = √2

3.5 = √12.25

√10 = √10

$$-\frac{5}{\sqrt{2}} = -\sqrt{\frac{25}{2}} = -\sqrt{12.5}$$

$$\frac{5}{2}\sqrt{3} = \sqrt{\frac{25 \times 3}{4}} = \sqrt{\frac{75}{4}} = \sqrt{18.75}$$

Writing the above numbers in descending order

√18.75, √12.25, √10, √2, – √12.5

So we get

5/2 √3, 3.5, √10, √2, -5/√2

14. Find a rational number and an irrational number between √3 and √5.

Solution:

Let $(\sqrt{3})^2 = 3$ and $(\sqrt{5})^2 = 5$

(i) There exists a rational number 4 which is the perfect square of a rational number 2.

(ii) There can be much more rational numbers which are perfect squares.

(iii) We know that

One irrational number between √3 and √5 = ½ (√3 + √5) = (√3 + √5)/ 2

15. Insert three irrational numbers between 2√3 and 2√5, and arrange in descending order.

Solution:

Take the square

$(2\sqrt{3})^2 = 12$ and $(2\sqrt{5})^2 = 20$

So the number 13, 15, 18 lie between 12 and 20 between $(\sqrt{12})^2$ and $(\sqrt{20})^2$

√13, √15, √18 lie between 2√3 and 2√5

Therefore, three irrational numbers between

2√3 and 2√5 are √13, √15, √18 or √13, √15 and 3√2.

Here

√20 > √18 > √15 > √13 > √12 or 2√5 > 3√2 > √15 > √13 > 2√3

Therefore, the descending order: 2√5, 3√2, √15, √13 and 2√3.

16. Give an example each of two different irrational numbers, whose

(i) sum is an irrational number.

(ii) product is an irrational number.

Solution:

(i) Consider a = √2 and b = √3 as two irrational numbers

Here

a + b = √2 + √3 is also an irrational number.

(ii) Consider a = √2 and b = √3 as two irrational numbers

Here

ab = √2 √3 = √6 is also an irrational number.

17. Give an example of two different irrational numbers, a and b, where a/b is a rational number.

Solution:

Consider a = 3√2 and b = 5√2 as two different irrational numbers

Here

a/b = 3√2/5√2 = 3/2 is a rational number.

18. If 34.0356 is expressed in the form p/q, where p and q are coprime integers, then what can you say about the factorization of q?

Solution:

We know that

34.0356 = 340356/10000 (in p/q form)

= 85089/2500

Here

85089 and 2500 are coprime integers

So the factorization of q = 2500 = $2^2 \times 5^4$

2	2500
2	1250
5	625
5	125
5	25
5	5
	1

Is of the form ($2^m \times 5^n$)

Where m and n are positive or non-negative integers.

19. In each case, state whether the following numbers are rational or irrational. If they are rational and expressed in the form p/q, where p and q are coprime integers, then what can you say about the prime factors of q?

(i) 279.034

$(ii) 76.\overline{17893}$

(iii) 3.010010001…

(iv) 39.546782

(v) 2.3476817681…

(vi) 59.120120012000…

Solution:

(i) 279.034 is a rational number because it has terminating decimals

279.034 = 279034/1000 (in p/q form)

= 139517/500 (Dividing by 2)

We know that

Factors of 500 = 2 × 2 × 5 × 5 × 5 = $2^2 \times 5^3$

Which is of the form $2^m \times 5^n$ where m and n are positive integers.

$(ii) 76.\overline{17893}$

It is a rational number as it has recurring or repeating decimals

Consider x = $76.\overline{17893}$

= 76.17893 17893 17893 …..

100000x = 7617893.178931789317893…..

By subtraction

99999x = 7617817

x = 7617817/99999 which is of p/q form

We know that

Prime factor of 99999 = 3 × 3 × 11111

q has factors other than 2 or 5 i.e. $3^2 \times 11111$

3	99999
3	33333
	11111

(iii) 3.010010001….

It is neither terminating decimal nor repeating

Therefore, it is an irrational number.

(iv) 39.546782

It is terminating decimal and is a rational number

39.546782 = 39546782/1000000 (in p/q form)

= 19773391/500000

We know that p and q are coprime

Prime factors of q = $2^5 \times 5^6$

Is of the form $2^m \times 5^n$ where m and n are positive integers

2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(v) 2.3476817681…

Is neither terminating nor repeated decimal

Therefore, it is an irrational number.

(vi) 59.120120012000….

It is neither terminating decimal nor repeated

Therefore, it is an irrational number.